LEC - 7

TRIGONOMETRIC EQUATIONS:

$$x + 2 = 4$$
$$-2 \quad -2$$
$$\boxed{x = 2}$$

$$x^2 - 4 = 0 \quad [\text{Quadratic}]$$
$$+4 \quad +4$$
$$x^2 = 4$$
$$\Rightarrow \boxed{x = \pm 2}$$

* We will study trig. equations,
  - → solve it using Calculator
  - → Solve Equations in Quadratic form
  - → Solve equations using fundamental identities.

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* Determine whether $\theta = \frac{\pi}{4}$ is a solution of equation $2\sin\theta - 1 = 0$?

⇒ Replace θ by $\frac{\pi}{4}$ in the equation $2\sin\theta - 1 = 0$

RESULT:

$$2\sin\left(\frac{\pi}{4}\right) - 1 = 0$$

$$\Rightarrow \cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} - 1 = 0$$

$$\Rightarrow \underbrace{\left(\sqrt{2} - 1\right)}_{\text{Is this } 0\text{?}} \neq 0$$

NO:

Therefor $\theta = \frac{\pi}{4}$ is NOT a solution.

Q: Is $\theta = \frac{\pi}{6}$, a solution of $2\sin\theta - 1 = 0$?

Replace θ by $\pi/6$

$$2\sin\theta - 1 = 0$$

$$\Rightarrow 2\sin\left(\frac{\pi}{6}\right) - 1 = 0$$

$$\Rightarrow 2 \cdot \frac{1}{2} - 1 = 0$$

$$\Rightarrow 1 - 1 = 0$$

$$\Rightarrow 0 = 0 \quad \text{(TRUE)}$$

Therefore $\theta = \frac{\pi}{6}$ is a solution of eq^n: $2\sin\theta - 1 = 0$.

Note: The equation $2\sin\theta - 1 = 0$ has other solutions besides $\theta = \frac{\pi}{6}$.

for example: $\theta = \frac{5\pi}{6}$ is a solution.

What about $\theta = \frac{13\pi}{6}$ .... [Check it yourself]

* Finding all the solutions of Trig. Equations:

- Solve the equation: $\cos\theta = \frac{1}{2}$

→ You need to find the general solution

you need to find a general formula for all solutions.

→ Period of cosine function is 2π.

In the interval $[0, 2\pi)$, there are 2 angles θ

for which $\cos\theta = \frac{1}{2}$ ($\theta = \frac{\pi}{3}$, $\theta = \frac{5\pi}{3}$)

Because

"Period of cosine function is 2π."

$$\theta = \frac{\pi}{3} + (2\pi)k \quad / \quad \theta = \frac{5\pi}{3} + (2\pi)k$$

where k is any integer

Find 8 solutions:

$k = -1$

$\Rightarrow \theta = \frac{\pi}{3} - 2\pi = \frac{-5\pi}{3}$, $\theta = \frac{5\pi}{3} - 2\pi = \frac{-\pi}{3}$

$k = 0 \Rightarrow \theta = \pi/3$, $\theta = \frac{5\pi}{3}$

$k = 1 \Rightarrow \theta = \frac{7\pi}{3}$; $\theta = \frac{5\pi}{3} + 2\pi = \frac{11\pi}{3}$

$k = 2$.

Solving a linear trig. Equation:

$2X + 3 = 5$

LINEAR.

* Solve the equation: $2\sin\theta + \sqrt{3} = 0 \quad (0 \le \theta < 2\pi)$

$X \Rightarrow 2X + \sqrt{3} = 0$

linear

Solution:

$$2\sin\theta + \sqrt{3} = 0$$

$$-\sqrt{3} \qquad -\sqrt{3}$$

$$2\sin\theta = -\sqrt{3}$$

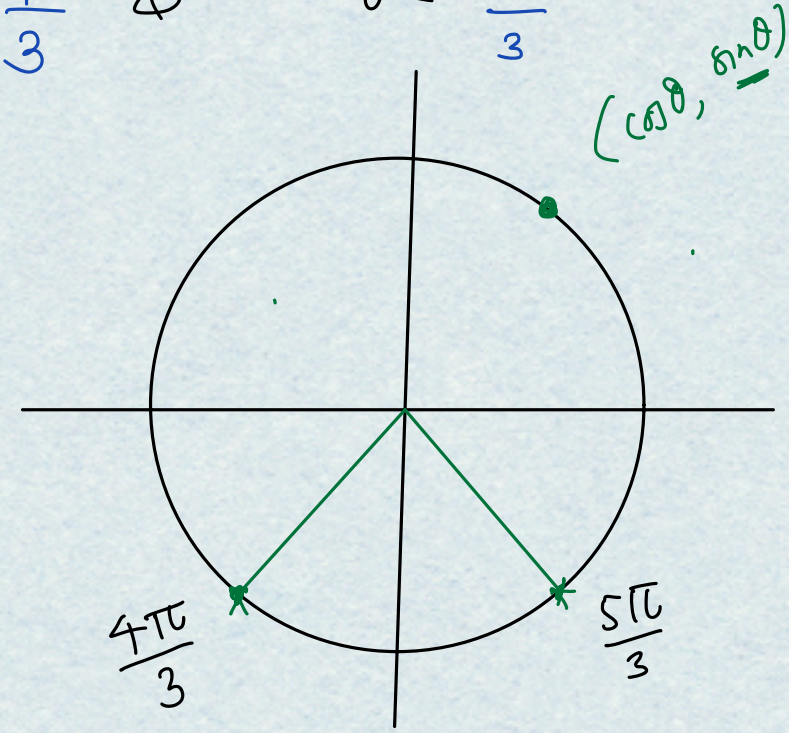
$$\Rightarrow \sin\theta = -\frac{\sqrt{3}}{2}$$ meaning

In the interval $[0, 2\pi)$, there are two angles θ for which $\sin\theta = -\frac{\sqrt{3}}{2}$.

$$\theta = \frac{4\pi}{3} \quad \& \quad \theta = \frac{5\pi}{3}$$

So, the solution for the equation: $2\sin\theta + \sqrt{3} = 0$ in $[0, 2\pi)$ are:

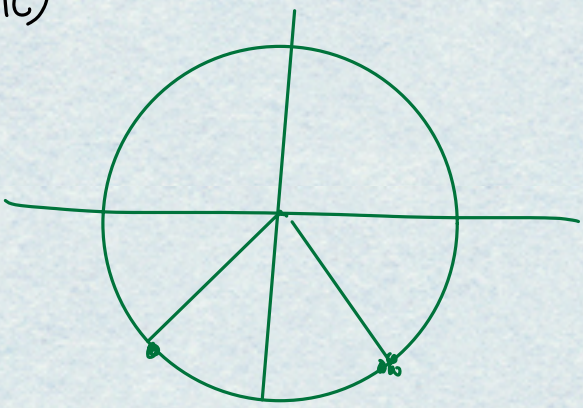
$$\left\{ \frac{4\pi}{3}, \frac{5\pi}{3} \right\}.$$



* $2\sin\theta + 5 = 4 \quad (0 \le \theta < 2\pi)$

→ Hint: $\sin\theta = -\frac{1}{2}$

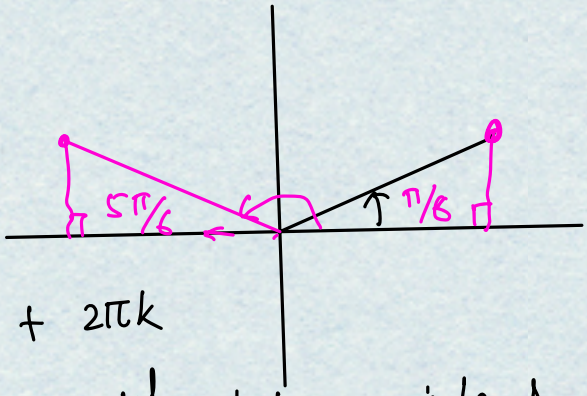
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* Solving trig. eqn. involving double angle:

. $\sin(2\theta) = \frac{1}{2}$; $0 \le \theta < 2\pi$

Solution: In the interval $[0, 2\pi)$, the sine function equals $\frac{1}{2}$ at $\Rightarrow \frac{\pi}{6}$ and $\frac{5\pi}{6}$



Then,

$$2\theta = \frac{\pi}{6} + 2\pi k \quad , \quad 2\theta = \frac{5\pi}{6} + 2\pi k$$

where k is any integer.

$$\Rightarrow \theta = \frac{\pi}{12} + \pi k \quad , \qquad \theta = \frac{5\pi}{12} + \pi k$$

Put: $k = -1$

$$\theta = \frac{\pi}{12} - \pi = -\frac{11\pi}{12} \qquad \theta = \frac{5\pi}{12} - \pi = -\frac{7\pi}{12}$$

$k = 0$

$$\theta = \frac{\pi}{12} \qquad \theta = \frac{5\pi}{12}$$

$k = 1$

$$\theta = \frac{\pi}{12} + \pi = \frac{13\pi}{12} \qquad \theta = \frac{5\pi}{12} + \pi = \frac{17\pi}{12}$$

$k = 2$

$$\theta = \frac{\pi}{12} + 2\pi = \frac{25\pi}{12} \qquad \theta = \frac{5\pi}{12} + 2\pi = \frac{29\pi}{12}$$

So, In the interval $[0, 2\pi)$, solutions for $\sin(2\theta) = \frac{1}{2}$ are $\left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$.

Q: $\sin(2\theta) = \frac{\sqrt{3}}{2}$; $0 \le \theta < 2\pi$

(Do yourself)

Ans: $\left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} \right\}$.

Solving trig. Eqn in Quadratic form:

Solve: $2\sin^2\theta - 3\sin\theta + 1 = 0$; $0 \le \theta < 2\pi$.

Soln: → This equation is quadratic in $\sin\theta$.

If you let $\boxed{\sin\theta = x}$

You need to learn how to factor Quadratic Equation.

$\Rightarrow 2x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 2x - x + 1 = 0$

Product: 2
Sum/diff = -3
(-2) * (-1)
-2 - 1 = -3

$\Rightarrow 2x(x-1) - 1(x-1) = 0$

$(x-1)[2x-1] = 0$

$\Rightarrow$ Either $x-1 = 0$ or $2x-1 = 0$

Zero product Property:

$a \cdot b = 0$

Then: Either $a = 0$ or $b = 0$

$\Rightarrow \boxed{x = 1}$, $\frac{2x}{2} = \frac{1}{2}$

$\Rightarrow \boxed{x = 1/2}$

$\Rightarrow \sin\theta = 1$ ↓ $\boxed{\theta = \frac{\pi}{2}}$

$\sin\theta = \frac{1}{2}$ → $\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$

Solution: $\left\{ \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$.

* Solve: $\tan\left(\theta - \frac{\pi}{2}\right) = 1 \quad ; \ 0 \le \theta < 2\pi$

Solution:

$$\tan\left(\theta - \frac{\pi}{2}\right) = \tan\frac{\pi}{4}$$

$$\downarrow$$

$$\theta - \frac{\pi}{2} = \frac{\pi}{4} + k\pi$$

$$+\frac{\pi}{2} \qquad +\frac{\pi}{2}$$

$$\Rightarrow \quad \theta = \frac{2\cdot\pi}{2\cdot 2} + \frac{\pi}{4} + k\pi \quad ; k \text{ is an integer}$$

$$\Rightarrow \boxed{\theta = \frac{3\pi}{4} + k\pi} \quad ; k \text{ is any integer.}$$

But in the interval $[0, 2\pi)$,

$$\boxed{\theta = \frac{3\pi}{4}} \quad (k = 0)$$

$$\theta = \frac{3\pi}{4} + 1\cdot\pi \qquad (k = 1)$$

$$= \frac{3\pi}{4} + \frac{4}{4}\pi$$

$$\boxed{\theta = \frac{7\pi}{4}}$$

are the only solutions.

Solve by using Calculator:

Solve: $\tan\theta = -2 \quad ; 0 \le \theta < 2\pi$.

Express solution rounded to two decimal places (radian)

$\Rightarrow \tan\theta = -2$

$\Rightarrow \theta = \tan^{-1}(-2) = -1.107148718$

≈ -1.11 (after rounding to 2 decimal places)

* $\tan(\theta + \pi) = \tan\theta$

$\Rightarrow \tan(-1.11) = \tan(-1.11 + \pi)$

$= \tan(2.03)$

$\theta_1 = 2.03$

$\tan(\theta + \pi + \pi) = \tan(\theta + 2\pi)$

$= \tan(-1.11 + 2\pi)$

≈ 5.17 radian

$\theta_2 = 5.17$ rad.