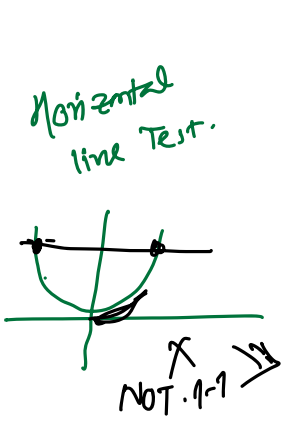
Lecture 6
The Inverse Sine, Cosine, and Tangent Functions



$f(x) \xrightarrow{\text{Inverse}} f^{-1}(x)$

- ONE-TO-ONE function:

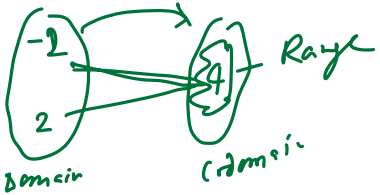
$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$



$x_1 \neq x_2$

$x_1^2 = x_2^2$

$y = f(x) = x^2$



Objectives

1. Find the Exact Value of an Inverse Sine, Cosine, or Tangent Function
2. Find an Approximate Value of an Inverse Sine Function
3. Use Properties of Inverse Functions to Find Exact Values of Certain Composite Functions
4. Find the Exact Value of Expressions Involving Inverse Trigonometric Functions

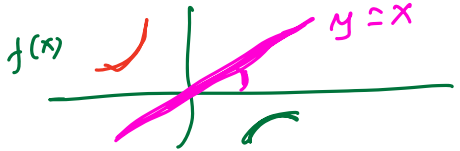
Review: One-to-One Functions and Inverses

$f^{-1}(f(x)) = x$: for all x in the domain of f $\quad f \quad f^{-1}$

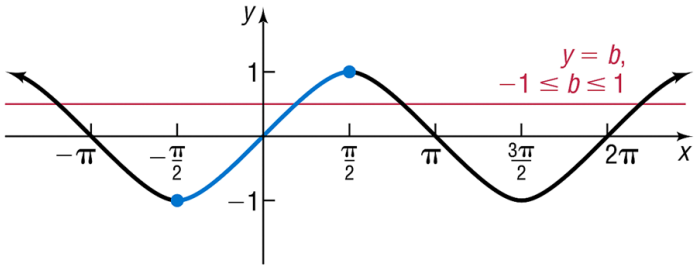
For a one-to-one function f and its inverse f^{-1}:

1. $f^{-1}(f(x)) = x$ for every x in the domain of f, and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}. CANCELLATION
2. The domain of f = the range of f^{-1}, and the range of f = the domain of f^{-1}.
3. The graphs of f and f^{-1} are reflections of one another about the line $y = \underline{\quad x \quad}$.
4. If $y = f(x)$ has an inverse function, the implicit equation of the inverse is $x = \underline{\quad f(y) \quad}$. Solving for y gives the explicit equation $y = \underline{\quad f^{-1}(x) \quad}$.

Key idea for this lecture: sine, cosine, and tangent are *not* one-to-one on their full domains, so we must *restrict* the domain before an inverse function can exist.



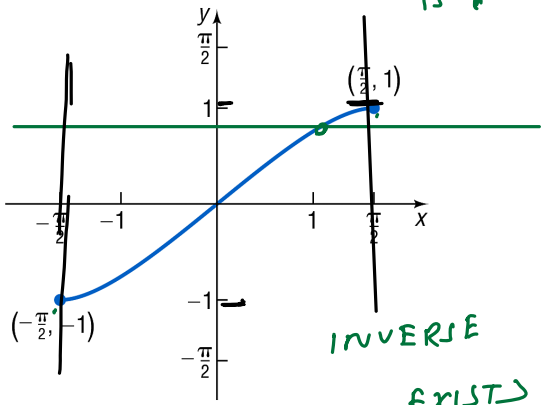
The Sine Function — Restricting the Domain



$y = \sin x, -\infty < x < \infty,$
$-1 \le y \le 1$

INVERSE DNE.

Now: $y = \sin x$ is 1-1



INVERSE EXISTS

$y = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}, -1 \le y \le 1$

Special attention: on $-\infty < x < \infty$, a horizontal line can cross $y = \sin x$ many times — not one-to-one. Restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the sine function *is* one-to-one, so an inverse exists there.

Definition 1: The Inverse Sine Function

$f(x) = \sin x$

$f^{-1}(x) = \sin^{-1} x$ $\left(\neq (\sin x)^{-1}\right)$

$\Downarrow$

$\frac{1}{\sin x}$

$y = \sin^{-1} x$ means $x = \sin y$

where

domain

$-1 \le x \le 1$ and $\left(-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right)$

Read it as: $\sin^{-1} x$ (also written $\arcsin x$) is *the angle whose sine is* x, always chosen from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\sin x$

values are: $[-1, 1]$

$\sin^{-1} x$

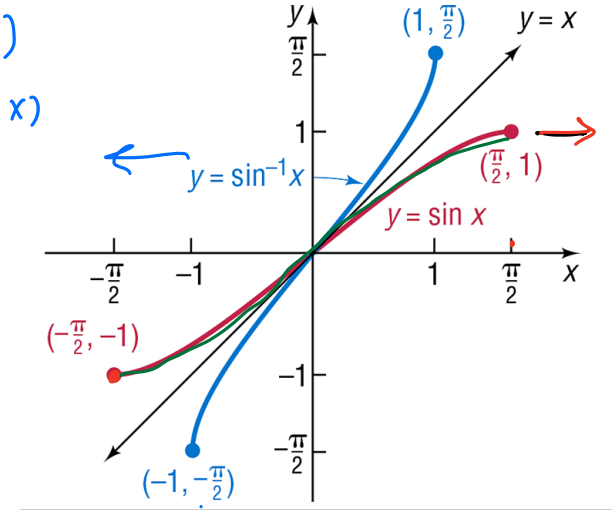
Angle

$y = \sin^{-1} x \Leftrightarrow x = \sin y$

The Inverse Sine Function — Graph

$\text{Dom}(y) = \text{Dom}(\sin^{-1} x)$
$= \text{Range}(\sin x)$
$= [-1, 1]$

$\text{Range}(\sin^{-1} x)$
$= \text{Dom}(\sin x)$
$= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$y = \sin x$
$\text{Dom}(y) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\text{Range}(y) = [-1, 1]$

$$y = \sin^{-1} x,\ -1 \le x \le 1,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

The graphs of $y = \sin x$ (restricted) and $y = \sin^{-1} x$ are reflections of each other about the line $y = x$.

$-\frac{\pi}{2}, -\frac{\pi}{3}, -\frac{\pi}{4}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \Rightarrow \theta$

$-1, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{2}}{2}, -\frac{1}{2}, 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1 \Rightarrow \sin\theta$

$\sin(-\theta) = -\sin\theta$

* $y = \sin^{-1} x \rightarrow$ Angle

$\text{Dom}(y) = [-1, 1]$

$\text{Range}(y) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

Objective 1

Find the Exact Value of an Inverse Sine Function

$-\frac{\pi}{2} \leq \sin^{-1}(-1) \leq \frac{\pi}{2}$

value should in angle

$-\pi/2$

Example: Exact Value of an Inverse Sine Function

Find the exact value of: $\sin^{-1}(-1)$. $= -\pi/2$

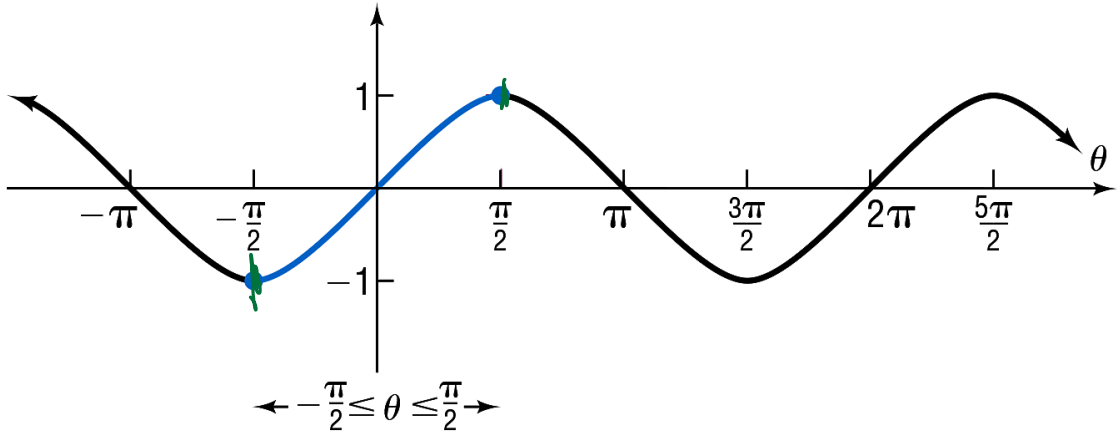
Step 1: Let $\theta = \sin^{-1}(-1)$. Then θ is the angle, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, whose sine equals -1:

$$\sin\theta = -1 \qquad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

Step 2: Look at the table of standard values.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Example: Exact Value of an Inverse Sine Function (cont.)



Step 3: The only angle θ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is -1 is $-\pi/2$. Therefore,

$$\sin^{-1}(-1) = -\pi/2$$

Example: Exact Value of an Inverse Sine Function

Find the exact value of: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$. $\Rightarrow$ $\theta = \sin^{-1}\left(\sqrt{3}/2\right) \Rightarrow \frac{\pi}{3}$

Solution: Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$. Then θ is the angle, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals $\frac{\sqrt{3}}{2}$:

$$\sin\theta = \frac{\sqrt{3}}{2} \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

Special attention: scan the standard-values table for $\frac{\sqrt{3}}{2}$ in the row $\sin\theta$ — it appears at exactly one θ inside the allowed interval.

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

Objective 2

Find an Approximate Value of an Inverse Sine Function

Example: Approximate Value of an Inverse Sine Function

Find an approximate value of each function. Express the answer in radians, rounded to two decimal places.

(a) $\sin^{-1}\frac{1}{3}$ (b) $\sin^{-1}\left(-\frac{1}{4}\right)$

Special attention: set your calculator to **radian** mode before evaluating — these are not "nice" angles like the earlier examples, so a decimal approximation is the only way to express them.

(a) $\sin^{-1}\frac{1}{3} \approx$ 0.34 (b) $\sin^{-1}\left(-\frac{1}{4}\right) \approx$ -0.25

Objective 3

Use Properties of Inverse Functions to Find Exact Values of Certain Composite Functions

$f^{-1}(f(x)) = x$ for all x in domain of f

$f(x)$

$\Rightarrow \sin^{-1}(\sin x) = x$ for all x in domain of $\sin x$

Similarly: $\sin(\sin^{-1} x) = x$ for all x in domain of $\sin^{-1} x$.

Property 1 — Composite Functions with Sine

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \qquad \text{where } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \qquad \text{where } -1 \le x \le 1$$

Special attention: these "cancellation" identities only work when x is already inside the stated interval. Outside that interval, you cannot simply cancel — see the next example.

$\sin^{-1}(\sin x) =$



$\sin^{-1}(\sin x) = x$ if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Example: Composite Functions — Direct Cancellation

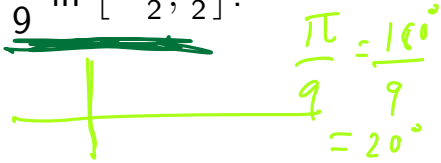
Find the exact value of $\sin^{-1}\left(\sin\frac{\pi}{9}\right)$.

$= \frac{\pi}{9}$

$\frac{\pi}{9}$ (lies / does not lie) in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solution: This has the form $\sin^{-1}(\sin x)$ with $x = \frac{\pi}{9}$. Check: is $\frac{\pi}{9}$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?

YES



Since it is, Property 1 applies directly:

$$\sin^{-1}\left(\sin\frac{\pi}{9}\right) = \frac{\pi}{9}$$

Example: Composite Functions — Outside the Domain

Find the exact value, if any, of: $\sin\left[\sin^{-1}(-2.5)\right]$.

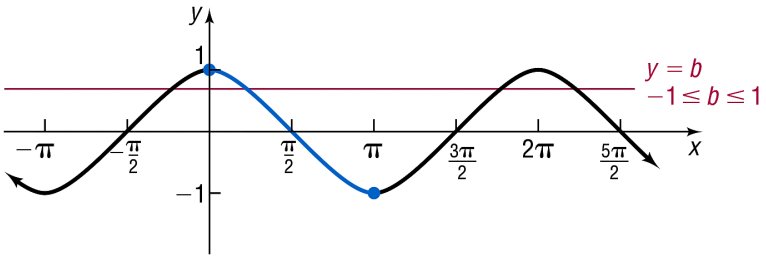
$\sin(\sin^{-1}(x)) = x$ if

Special attention: this has the form $\sin(\sin^{-1} x)$ with $x = -2.5$. Before applying Property 1, check that x is in the *domain of* $\sin^{-1}$, which is $[-1, 1]$.

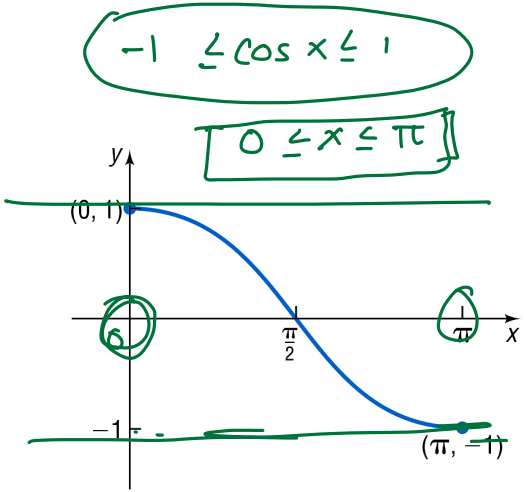
Is $-2.5 \in [-1, 1]$? NO

Therefore $\sin^{-1}(-2.5)$ is NOT DEFINED, and so $\sin\left[\sin^{-1}(-2.5)\right]$ is also undefined.

The Cosine Function — Restricting the Domain



$y = \cos x,\ -\infty < x < \infty,\ -1 \le y \le 1$



$y = \cos x,\ 0 \le x \le \pi,\ -1 \le y \le 1$

Restricted to $[0, \pi]$, the cosine function is one-to-one, so an inverse exists there.

Definition 2: The Inverse Cosine Function

$$y = \cos^{-1} x \quad \text{means} \quad x = \cos y$$

where

$$-1 \leq x \leq 1 \qquad \text{and} \qquad 0 \leq y \leq \pi$$

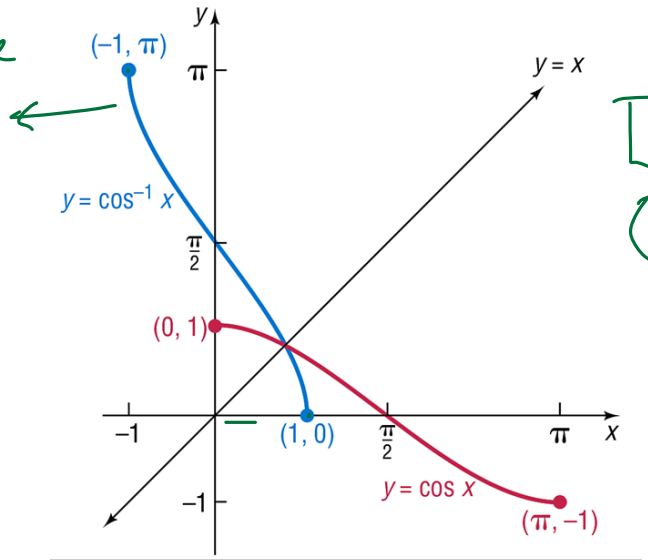
Special attention: the output interval for $\cos^{-1}$ is $[0, \pi]$ — *different* from the $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ used for $\sin^{-1}$. Mixing these up is the most common error with inverse trig.

The Inverse Cosine Function — Graph

$\cos^{-1} x$

$0 \le \cos^{-1} x \le \pi$ range

$-1 \le x \le 1$ domain



$y = \cos^{-1} x,\ -1 \le x \le 1,\ 0 \le y \le \pi$

$\cos x$

$-1 \le \cos x \le 1$ Range

$0 \le x \le \pi$ Domain

The graphs of $y = \cos x$ (restricted to $[0, \pi]$) and $y = \cos^{-1} x$ are reflections of each other about $y = x$.

Example: Exact Value of an Inverse Cosine Function

Find the exact value of: $\cos^{-1}\frac{1}{2}$. $\Rightarrow$ $\theta = \cos^{-1}(1/2)$, what value of θ gives $\cos\theta = 1/2$

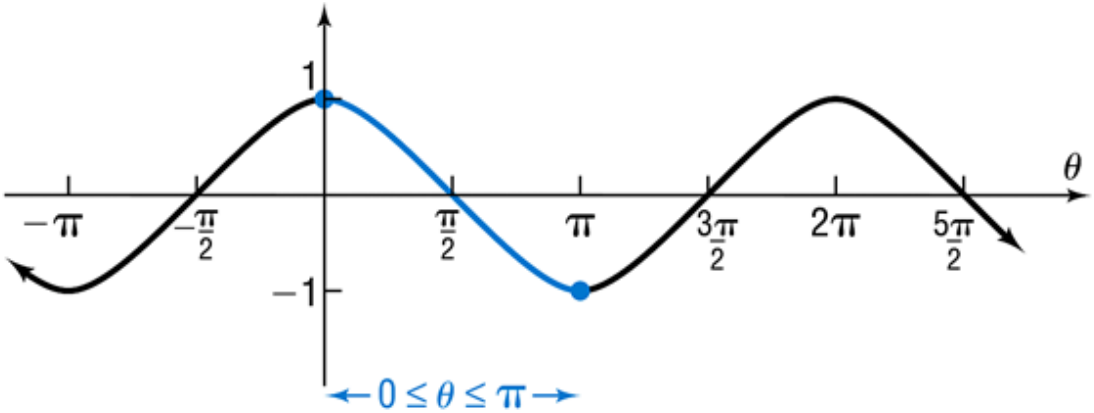
Step 1: Let $\theta = \cos^{-1}\frac{1}{2}$. Then θ is the angle, $0 \le \theta \le \pi$, whose cosine equals $\frac{1}{2}$:

$$\cos\theta = \frac{1}{2} \qquad 0 \le \theta \le \pi$$

θ	$\cos\theta$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$
π	-1

Step 2: Look at the table of standard values.

Example: Exact Value of an Inverse Cosine Function (cont.)



Step 3: The only angle θ in $[0, \pi]$ whose cosine is $\frac{1}{2}$ is $\pi/3$. Therefore,

$$\cos^{-1}\frac{1}{2} = \pi/3$$

Example: Exact Value of an Inverse Cosine Function

Find the exact value of: $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

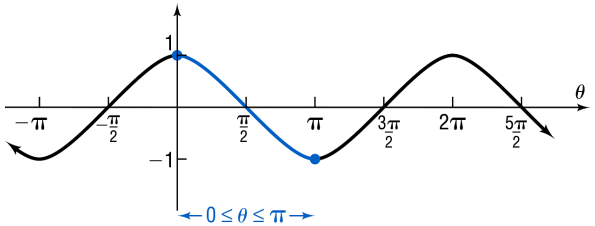
Solution: Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Then θ is the angle, $0 \le \theta \le \pi$, whose cosine equals $-\frac{\sqrt{3}}{2}$:

$$\cos\theta = -\sqrt{3}/2 \qquad 0 \le \theta \le \pi$$

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

$* \boxed{\cos(-\theta) = \cos\theta} *$

Cosine EVEN function



$\frac{5\pi}{6} \in [0, \pi]$ (True)

Property 2 — Composite Functions with Cosine

(2)

$$\begin{cases} f^{-1}(f(x)) = \cos^{-1}(\cos x) = x & \text{where } 0 \le x \le \pi \\ f(f^{-1}(x)) = \cos(\cos^{-1} x) = x & \text{where } -1 \le x \le 1 \end{cases}$$

for all x in the domain of $\cos x$

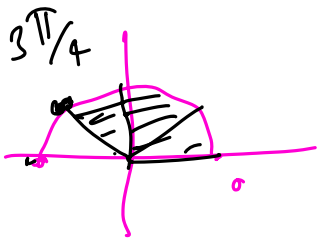
for all x in the domain of $\cos^{-1} x$

Example: Cosine Composite — Special Attention

Find the exact value of (a) $\cos^{-1}\left(\cos\frac{\pi}{15}\right)$ (b) $\cos^{-1}\left[\cos\left(-\frac{3\pi}{4}\right)\right]$

(a) Is $\frac{\pi}{15}$ in $[0,\pi]$? YES So Property 2 applies directly:

$$\cos^{-1}\left(\cos\frac{\pi}{15}\right) = \frac{\pi}{15}$$

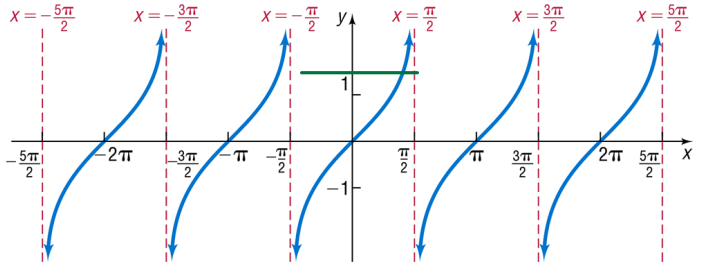


(b) Careful: $-\frac{3\pi}{4}$ is *not* in $[0,\pi]$, so Property 2 cannot be used directly. But cosine is **even**:

$\cos(-x) = \cos x$, so rewrite $\cos\left(-\frac{3\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$, which *is* in $[0,\pi]$. Now apply Property 2:

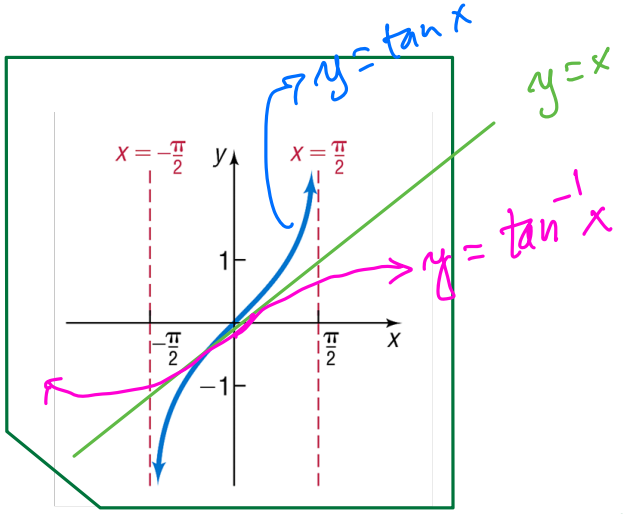
$$\cos^{-1}\left[\cos\left(-\frac{3\pi}{4}\right)\right] = \cos^{-1}\left[\cos\left(\frac{3\pi}{4}\right)\right] = \frac{3\pi}{4}$$

The Tangent Function — Restricting the Domain



$y = \tan x,\ -\infty < x < \infty,\ x$ not equal to odd multiples of $\frac{\pi}{2},\ -\infty < y < \infty$

Asymptotes Domain $\tan x$: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$y = \tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2},\ -\infty < y < \infty$ RANGE

Restricted to the open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the tangent function is one-to-one, so an inverse exists there.

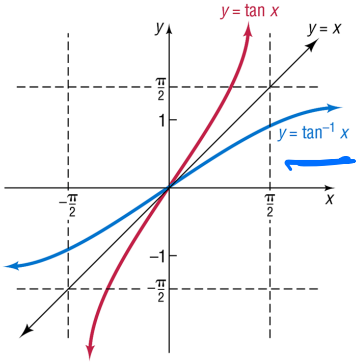
Domain

Definition 3: The Inverse Tangent Function

$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y$$

where

$$-\infty < x < \infty \qquad \text{and} \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$$



Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

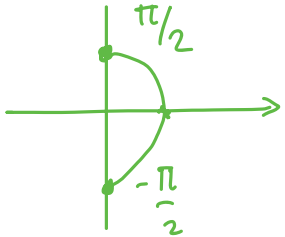
$-\frac{\pi}{2}, \frac{\pi}{2}$ are NOT included

Example: Exact Value of an Inverse Tangent Function

Find the exact value of: $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$.

Solution: Let $\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$. Then θ is the angle, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, whose tangent equals $-\frac{\sqrt{3}}{3}$:

$$\tan\theta = \frac{-\sqrt{3}}{3} \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$



$\tan(-\theta) = -\tan\theta$

$\tan(\pi/6) = \frac{\sqrt{3}}{3}$

$$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \frac{-\pi}{6}$$

$\tan\left(-\frac{\pi}{6}\right) = -\tan(\pi/6)$

$= -\frac{\sqrt{3}}{3}$

Property 3 — Composite Functions with Tangent

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \qquad \text{where } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \qquad \text{where } -\infty < x < \infty$$

Summary so far:

	Domain of f^{-1}	Range of f^{-1}	Restricted interval for f
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Objective 4

Find the Exact Value of Expressions Involving Inverse Trigonometric Functions

The Right-Triangle Method — Special Attention

Most Imp. Most Confusing

To evaluate an expression like $\sin\left[\tan^{-1}\frac{1}{3}\right]$ the argument is *not* a standard angle, so the table method from before will not work. Instead:

1. Let $\theta =$ the inner inverse function. Identify the *ratio* this gives you (e.g., $\tan\theta = \frac{1}{3}$ means opposite $= 1$, adjacent $= 3$).
2. Use the **range of the inverse function** to determine which *quadrant* θ lies in.
3. Find the third side with the Pythagorean theorem, $c^2 = a^2 + b^2$.
4. Read the desired ratio (sine, cosine, or tangent) directly off the triangle, fixing signs for the quadrant found in Step 2.

$\theta = \tan^{-1}\left(\frac{1}{3}\right) \Rightarrow \tan\theta = \frac{1}{3}$

Example: $\sin\left[\tan^{-1}\frac{1}{3}\right]$ (1 of 2)

θ

$\tan\theta = \frac{1}{3} = \frac{b}{a}$ (+ve)

Find the exact value of: $\sin\left[\tan^{-1}\frac{1}{3}\right]$. $\Rightarrow \sin\theta$

Step 1: Let $\theta = \tan^{-1}\frac{1}{3}$. Then $\tan\theta =$ $\frac{1}{3}$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

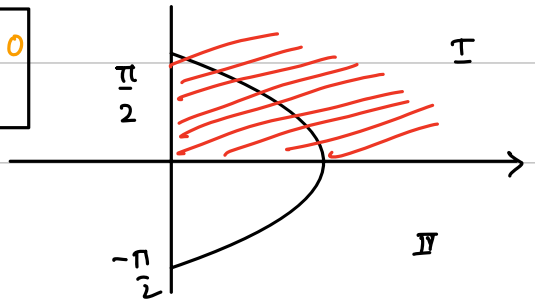
Step 2: We seek $\sin\theta$. Because $\tan\theta > 0$, it follows that $0 < \theta < \frac{\pi}{2}$, so θ lies in quadrant I.

$\frac{b}{a} = \tan\theta = \frac{1}{3} > 0$

$a = 3$

$b = 1$

$c = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$



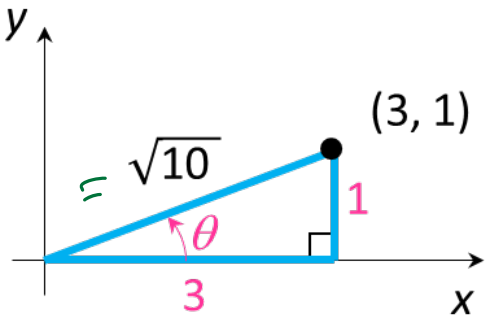
Example: $\sin\left[\tan^{-1}\frac{1}{3}\right]$ (2 of 2)

Since $\tan\theta = \frac{b}{a} = \frac{1}{3}$, take the side opposite θ as $b = 1$ and the side adjacent as $a = 3$. Find the hypotenuse:

$$c = \sqrt{a^2 + b^2} = \sqrt{9+1} = \sqrt{10}$$

$$\sin\theta = \frac{b}{c} = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\sin\left[\tan^{-1}\frac{1}{3}\right] = \frac{\sqrt{10}}{10}$$



Example: $\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (1 of 2)

$\sin\theta = -\frac{1}{2} < 0$

Find the exact value of: $\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$.

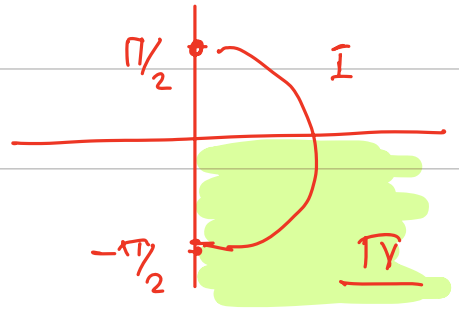
Step 1: Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$. Then $\sin\theta =$ $-\frac{1}{2}$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

Step 2: We seek $\cos\theta$. Because $\sin\theta < 0$, it follows that _____ $\le \theta <$ _____, so θ lies in quadrant IV.

$\sin\theta = -\frac{1}{2} = \frac{b}{c}$

$b = 1$

$c = 2$



Example: $\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (2 of 2)

$\sin\theta = -\frac{1}{2}$ for θ in quadrant IV, so opposite $= -1$, hypotenuse $= 2$. Find the adjacent side:

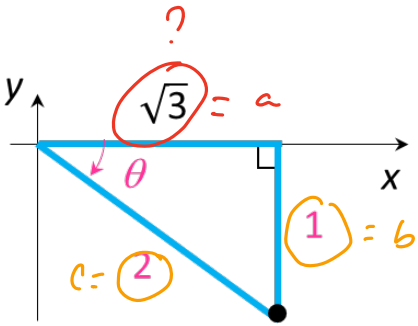
$$a = \sqrt{c^2 - b^2} = \sqrt{2^2 - 1^2} = \sqrt{4-1} = \sqrt{3}$$

$c^2 = a^2 + b^2$

$\Rightarrow a^2 = c^2 - b^2$

S | A

T | C



$$\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{a}{c} = \frac{\sqrt{3}}{2}$$

Example: $\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right]$ (1 of 2)

Find the exact value of: $\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right]$. $= \tan\theta$

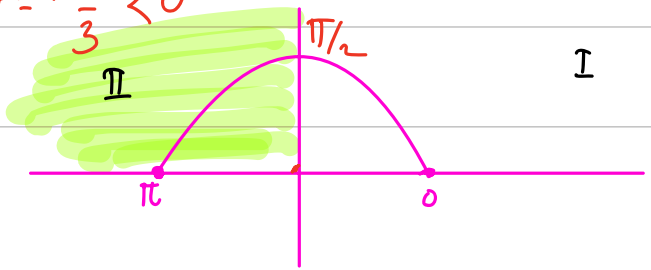
Step 1: Let $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$. Then $\cos\theta =$ $-\frac{1}{3}$, where $0 \leq \theta \leq \pi$.

Step 2: We seek $\tan\theta$. Because $\cos\theta < 0$, it follows that $\frac{\pi}{2} < \theta \leq \pi$, so θ lies in quadrant II.

$\cos\theta = -\frac{1}{3} < 0$

$\cos\theta = -\frac{1}{3} = \frac{a}{c}$

opposite = ?

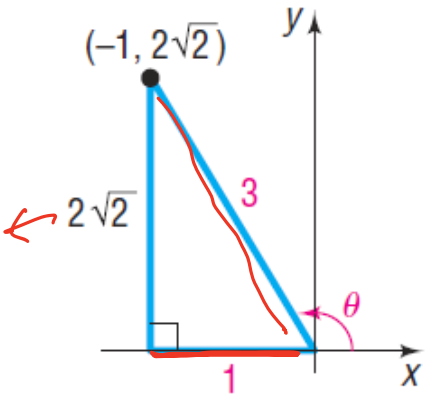


Example: $\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right]$ (2 of 2)

$\cos\theta = -\frac{1}{3}$ for θ in quadrant II, so adjacent $= -1$, hypotenuse $= 3$. Find the opposite side:

$$b = \sqrt{c^2 - a^2} = \sqrt{3^2 - 1^2} = \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$$

$$\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right] = \tan\theta = \frac{b}{a} = \frac{2\sqrt{2}}{1} = -2\sqrt{2}$$



$$\cos\theta = -\frac{1}{3}$$

Summary

- $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ each give **one** angle, chosen from a fixed restricted interval — memorize the three intervals.
- For *standard* angles, use the table of exact values.
- For a *calculator* value, always check radian mode.
- Composite-function properties ($f^{-1}(f(x)) = x$, etc.) cancel **only** when the input lies in the correct interval — otherwise rewrite the angle first (even/odd identities) or the expression is undefined.
- For an expression like $\sin\left[\tan^{-1}\frac{1}{3}\right]$: draw the right triangle, fix the quadrant from the *range* of the inverse function, then read off the ratio.

* $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

* $\cos^{-1}\left(\sin\frac{7\pi}{6}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

$\sin\frac{7\pi}{6} = -\frac{1}{2}$

* $\cos^{-1}\left(\cos\left(-\frac{2\pi}{3}\right)\right) = ?$

$= \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

$\cos^{-1}x$

$[0, \pi]$

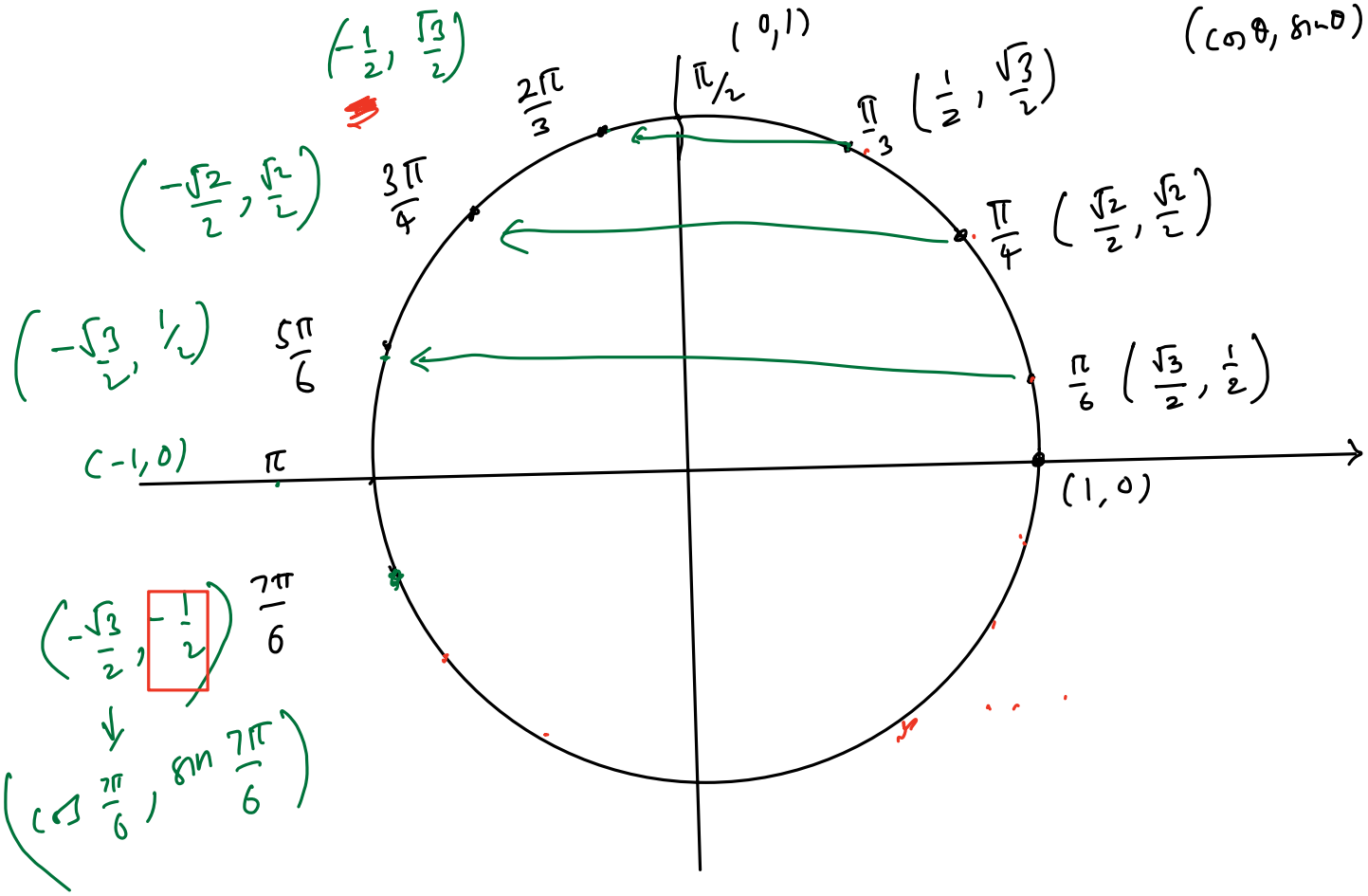
$0 \le \cos^{-1}x \le \pi$

$\frac{2\pi}{3}$ lies in $[0, \pi]$ Is $-\frac{2\pi}{3}$ in the interval $[0, \pi]$

* Since, cosine is Even function.

$\therefore \cos(-\theta) = \cos\theta$

$\cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$ $\left(\frac{2\pi}{3}\right)$


$(\cos\theta, \sin\theta)$
$(0,1)$
$\pi/2$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{3\pi}{4}$
$\frac{\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{5\pi}{6}$
$\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$(-1,0)$
π
$(1,0)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\frac{7\pi}{6}$
$\left(\cos\frac{7\pi}{6}, \sin\frac{7\pi}{6}\right)$