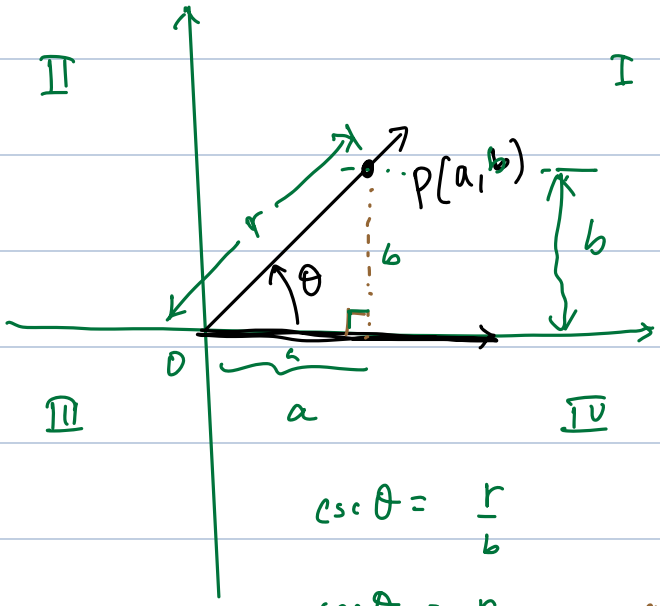
To find the Exact Values of Trigonometric function of any angle.

$\theta = 30^0, 45^0, 60^0$



$r^2 = a^2 + b^2$

$r = \sqrt{a^2+b^2}$

$\sin\theta = \frac{b}{r} = \frac{b}{\sqrt{a^2+b^2}}$

$r \neq 0$

$\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+b^2}}$

$r \neq 0$

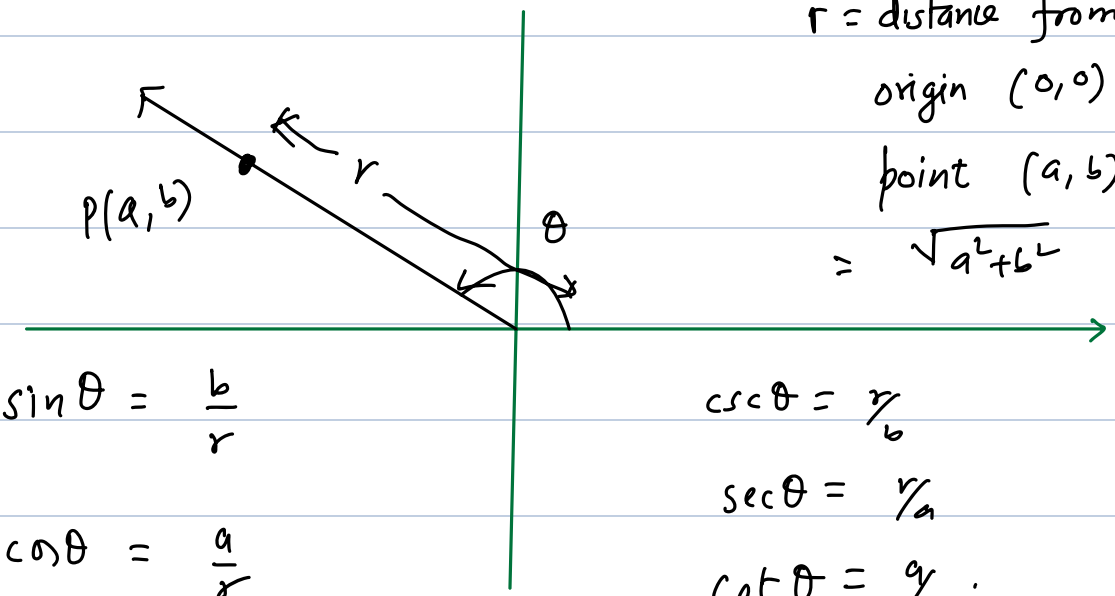
$\tan\theta = \frac{b}{a}$

$a \neq 0$

$\csc\theta = \frac{r}{b}$ $\quad b \neq 0$

$\sec\theta = \frac{r}{a}$ $\quad a \neq 0$

$\cot\theta = \frac{a}{b}$. $\quad b \neq 0$



r = distance from origin (0, 0) to the point (a, b).

$= \sqrt{a^2+b^2}$

$\sin\theta = \frac{b}{r}$

$\cos\theta = \frac{a}{r}$

$\tan\theta = b/a$

$\csc\theta = r/b$

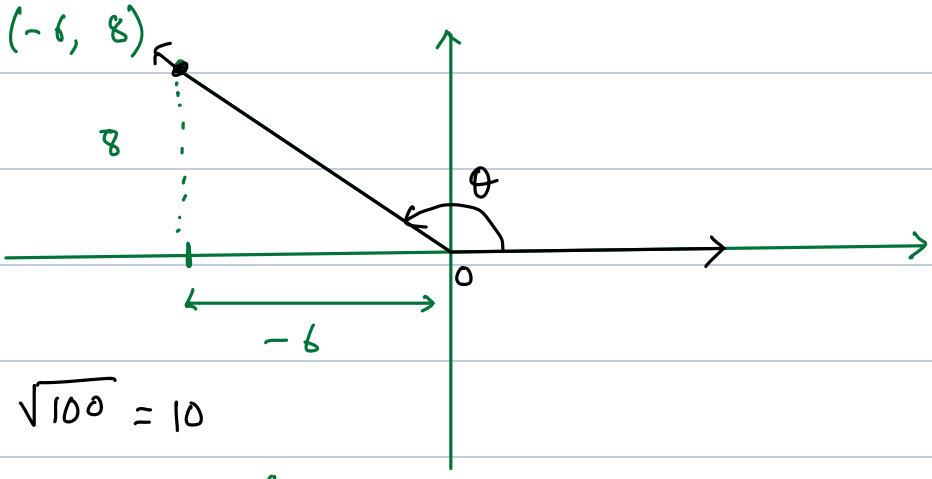
$\sec\theta = r/a$

$\cot\theta = a/b$.

Note: If denominator = 0, then ratio is UNDEFINED.

Example: Let P(−6, 8) is a point in terminal side. find the exact values of six trig. functions of +ve angle θ.

⇒ Point: P(−6, 8)



$a = -6$

$b = 8$

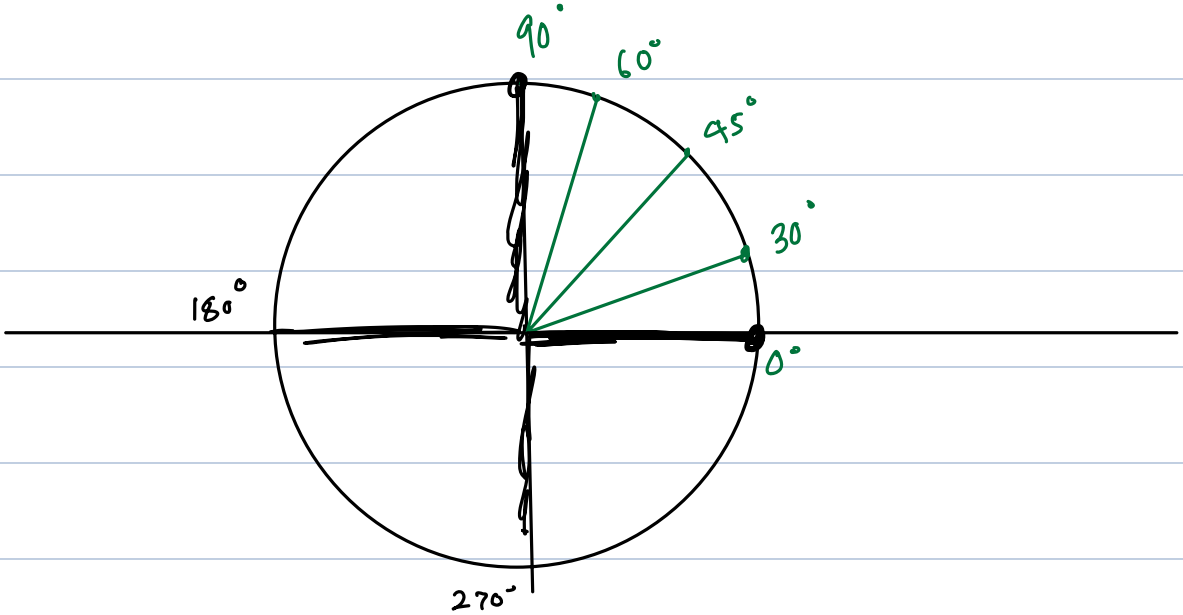
$r = \sqrt{(-6)^2 + 8^2}$

$= \sqrt{36 + 64} = \sqrt{100} = 10$

$\sin\theta = \frac{b}{r} = \frac{8}{10} = \frac{4}{5}$ $\quad \csc\theta = \frac{5}{4}$

$\cos\theta = \frac{a}{r} = \frac{-6}{10} = -\frac{3}{5}$ $\quad \sec\theta = -\frac{5}{3}$

$\tan\theta = \frac{b}{a} = \frac{8}{-6} = -\frac{4}{3}$ $\quad \cot\theta = -\frac{3}{4}$



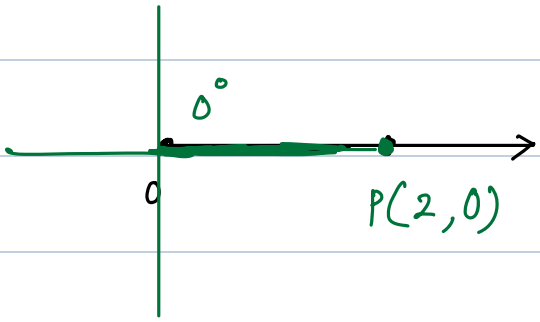
$\theta = 0^\circ = 0$ radian.

$a = 2$

$b = 0$

$r = \sqrt{a^2+b^2} = \sqrt{2^2+0^2} = \sqrt{4}$

$= 2$



$\sin 0^\circ = \frac{b}{r} = \frac{0}{2} = 0$

$\cos 0^\circ = \frac{a}{r} = \frac{2}{2} = 1$

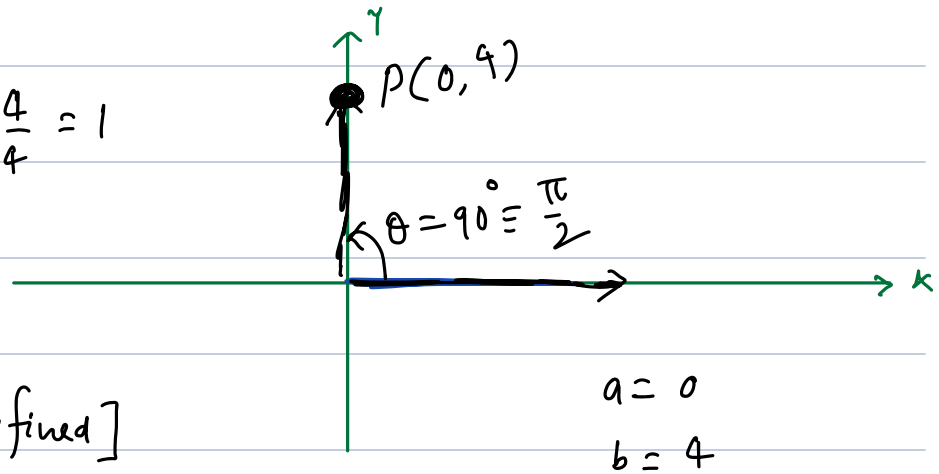
$\tan 0^\circ = \frac{b}{a} = \frac{0}{2} = 0$

$\csc 0^\circ = \frac{r}{b} = \frac{2}{0}$ [UNDEFINED]

$\sec 0^\circ = \frac{r}{a} = \frac{2}{2} = 1$

$\cot 0^\circ = \frac{a}{b} = \frac{2}{0}$ [UNDEFINED]

$\sin \frac{\pi}{2} = \sin 90^\circ = \frac{b}{r} = \frac{4}{4} = 1$



$\cos \frac{\pi}{2} = \frac{a}{r} = \frac{0}{4} = 0$

$\tan \frac{\pi}{2} = \frac{b}{a} = \frac{4}{0}$ [undefined]

$a = 0$

$b = 4$

$r = \sqrt{4^2+0^2}$

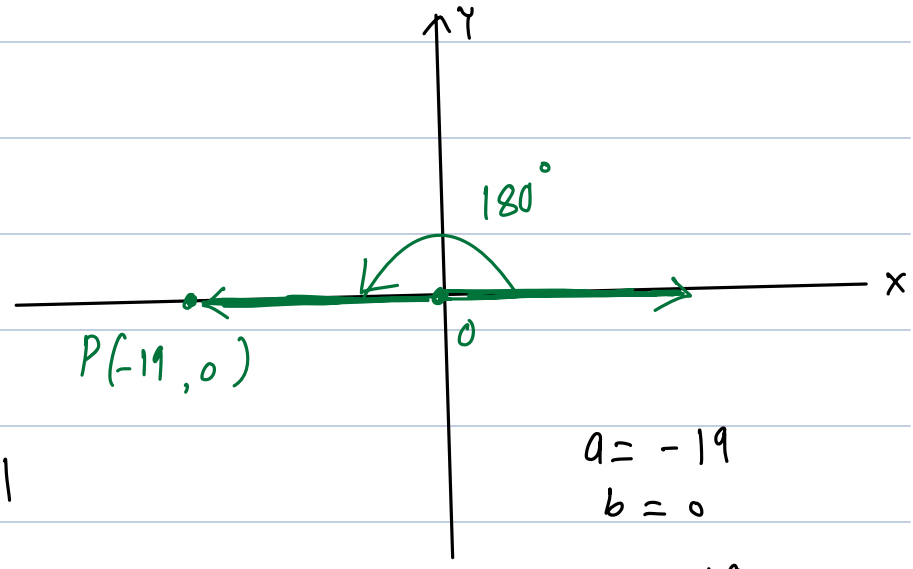
$= 4$

$\csc \frac{\pi}{2} = 1$

$\sec \frac{\pi}{2} = \frac{r}{a} = \frac{4}{0}$ [Undefined]

$\cot \pi/2 = \frac{a}{b} = \frac{0}{4} = 0$

$\theta = 180^\circ = \pi$



$a = -19$
$b = 0$
$r = 19$

$\sin \pi = \sin 180^\circ = \frac{b}{r} = \frac{0}{19} = 0$

$\cos 180^\circ = \frac{a}{r} = \frac{-19}{19} = -1$

$\tan \pi = \frac{b}{a} = \frac{0}{-19} = 0$

$\csc \pi = \frac{r}{b} = \frac{19}{0}$ [Undefined]

$\sec 180^\circ = \frac{r}{a} = \frac{19}{-19} = -1$

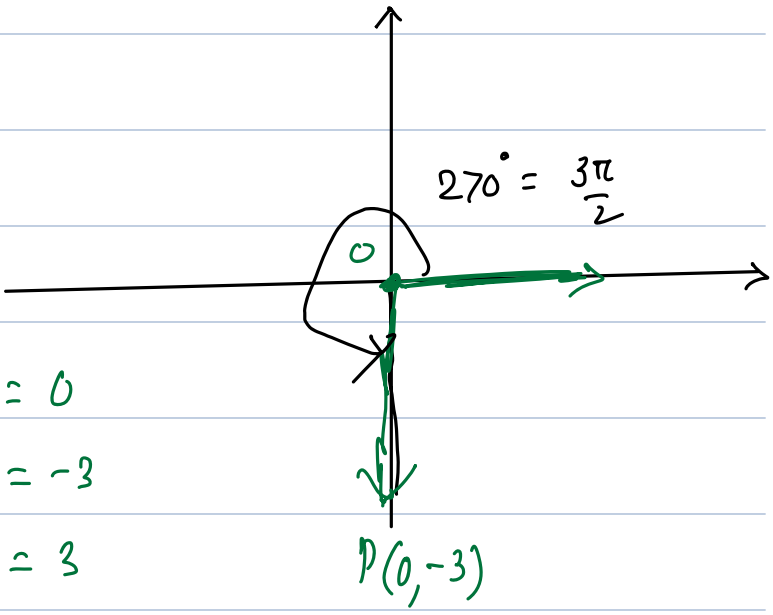
$\cot 180^\circ = \frac{a}{b} = \frac{-19}{0}$ [Undefined]

$\sin 270^\circ = \frac{b}{r} = \frac{-3}{3} = -1$

$\cos 270^\circ = \frac{a}{r} = \frac{0}{3} = 0$

$\tan 270^\circ = \frac{b}{a} = \frac{-3}{0}$

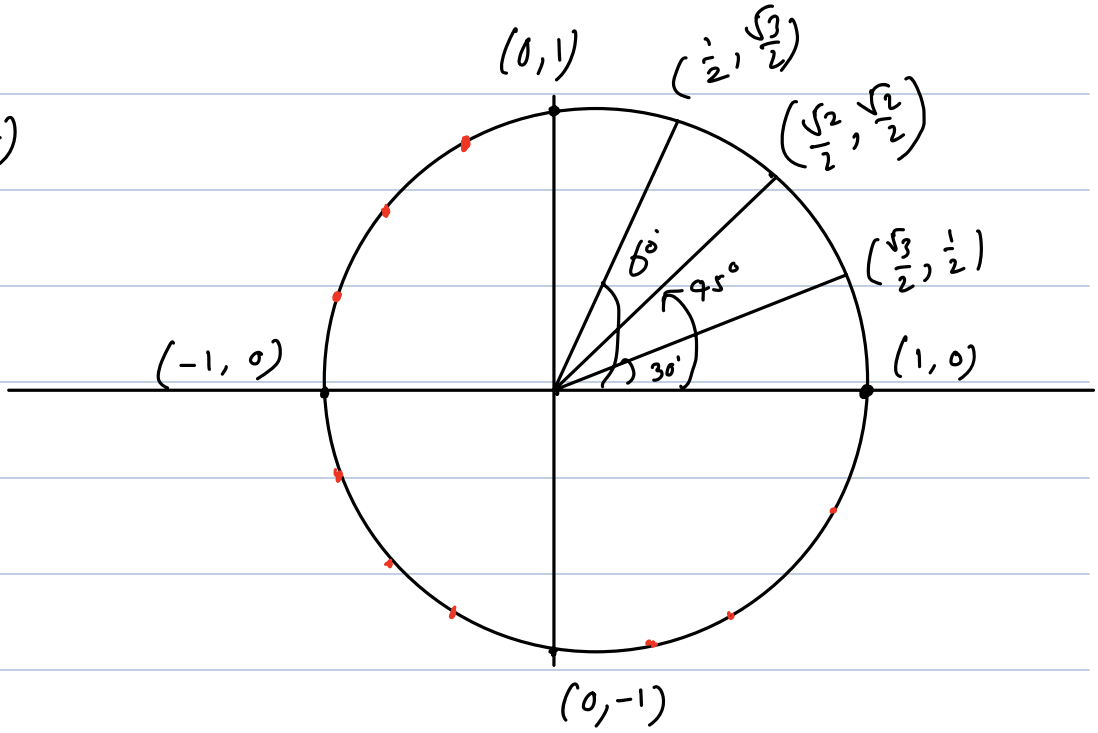
(UNDEFINED)



$a = 0$
$b = -3$
$r = 3$

$\cot 270^\circ = \cot \frac{3\pi}{2} = \frac{a}{b} = \frac{0}{-3} = 0$

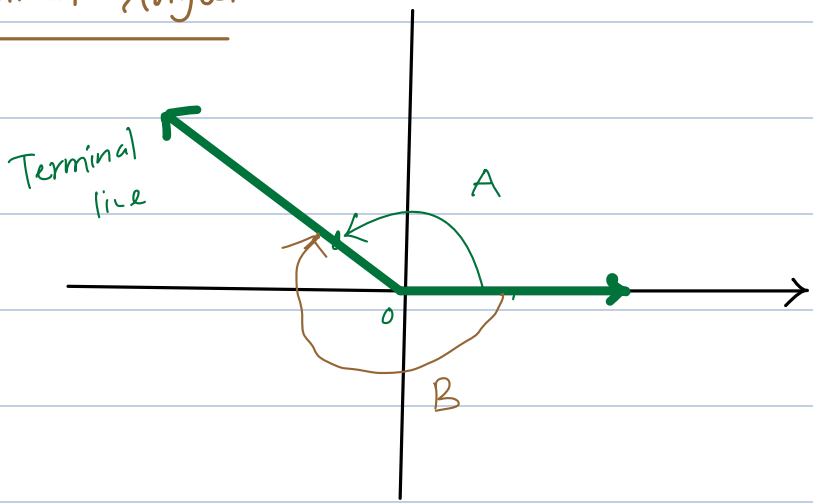
$P(\cos\theta, \sin\theta)$



Definition:

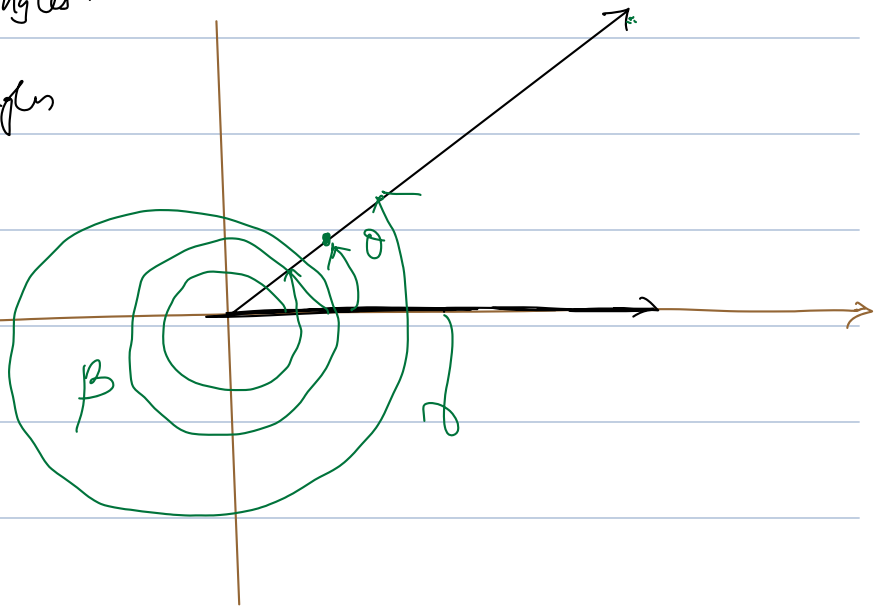
Co-terminal Angle:

The angles whose terminal sides are the same.



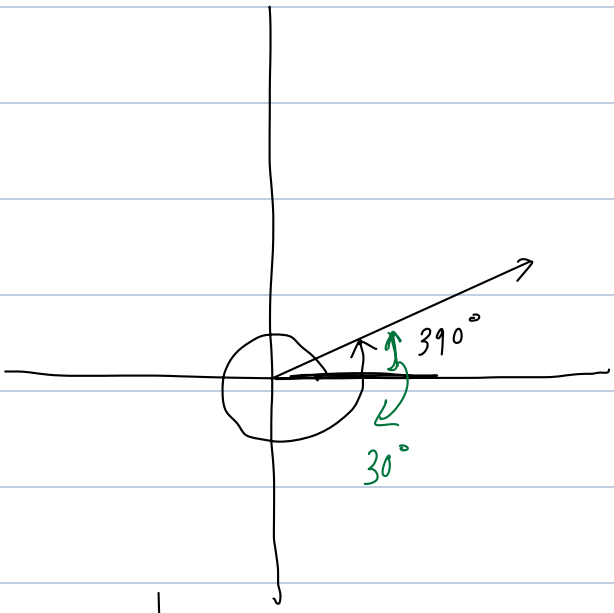
θ, β, γ are co-terminal angles.

Note: All the co-terminal angles have the same value of the trig. functions.

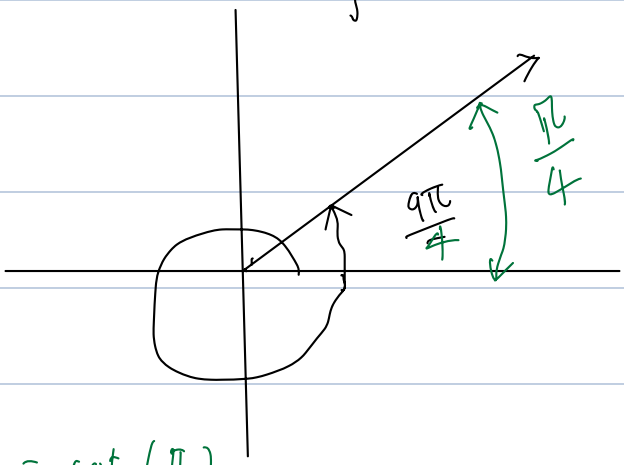


Q: Find the exact value of $\cos 390^\circ$.

$$\cos(390^\circ) = \cos(360^\circ + 30^\circ)$$
$$= \cos 30^\circ$$
$$= \frac{\sqrt{3}}{2}.$$



$$\cot\left(\frac{9\pi}{4}\right)$$
$$\downarrow$$
$$(9.45^\circ)$$
$$= 405^\circ$$
$$\underbrace{\qquad}_{45^\circ} \quad \text{coterminal}$$



$$\cot\left(\frac{9\pi}{4}\right) = \cot\left(2\pi + \frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right) = 1$$

$$\sin(\theta) = \sin(360^\circ + \theta) \Rightarrow \sin\theta = \sin(2\pi + \theta)$$

If k is any integer then:

$$\sin\theta = \sin(360^\circ \cdot k + \theta)$$
$$\cos\theta = \cos(360^\circ \cdot k + \theta)$$

$$\sin\theta = \sin(2\pi k + \theta)$$

Quadrant 's Rule:

A
(All positive)

S
II
I
(sinθ and cscθ are positive)

III
IV

T
(tanθ and cotθ are positive)

C
(cosine θ and secθ are positive)

Q: Name the quadrant:

- sinθ is negative and cosine θ is positive

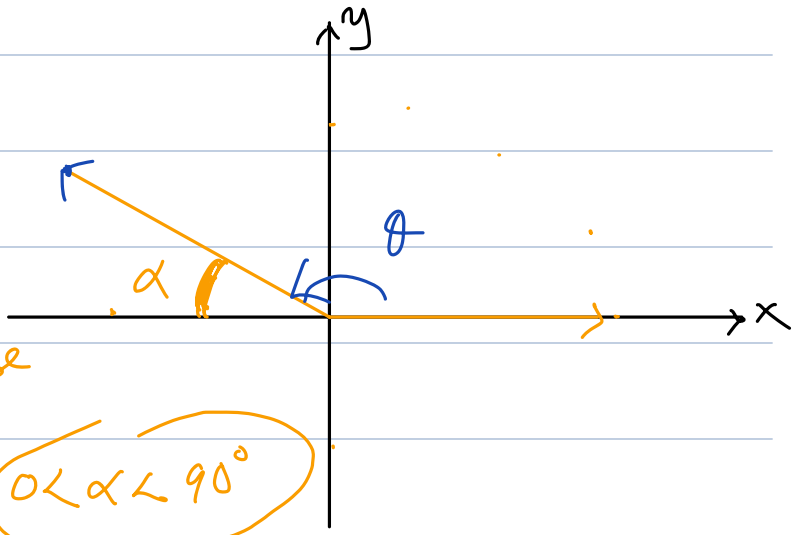
→ IV

sinθ < 0

- sinθ < 0 & cosθ < 0

⇒ III :

Reference Angle:



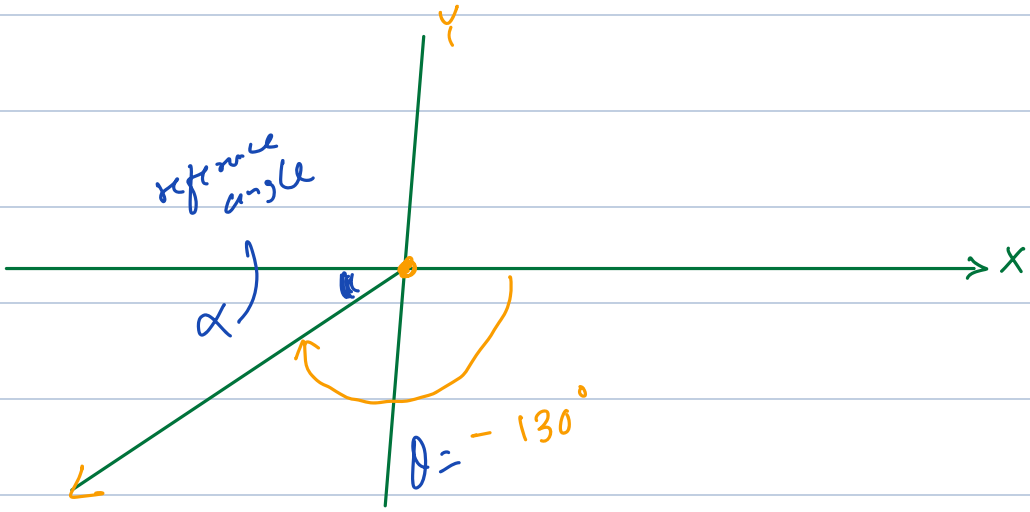
θ = might be acute/obtuse

α = must be acute: $0 < \alpha < 90^\circ$

Reference angle.

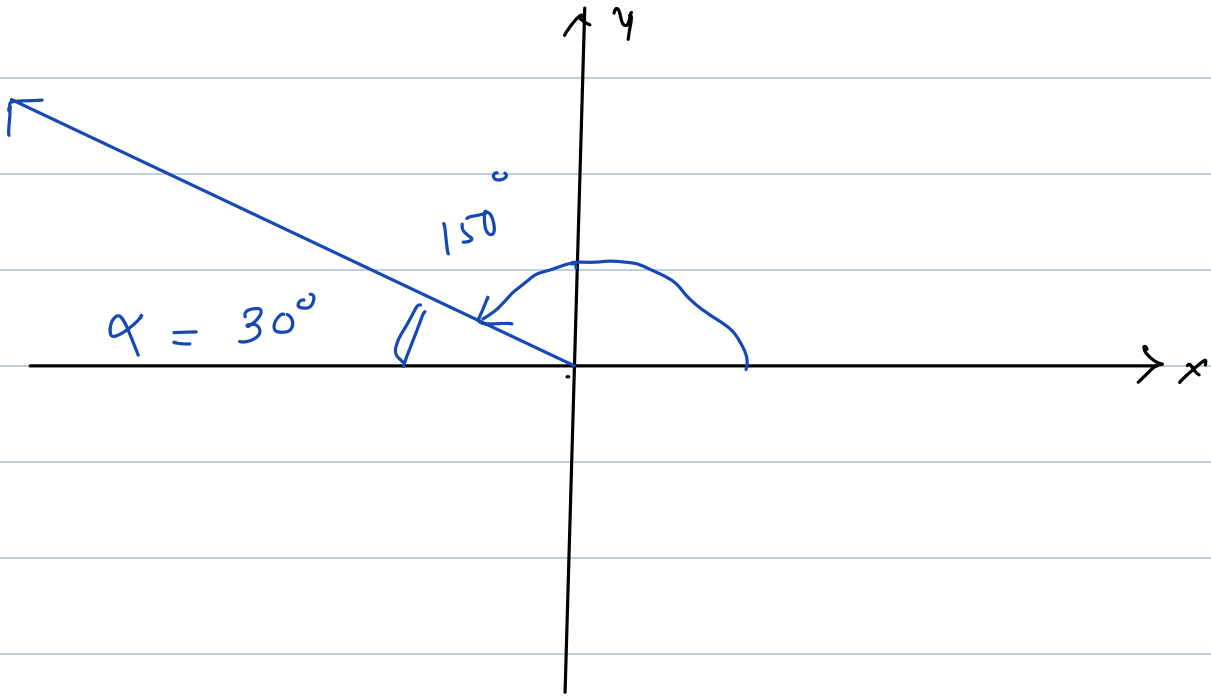
Angle bet^{n}: X-axis and terminal side

NOT Y-axis

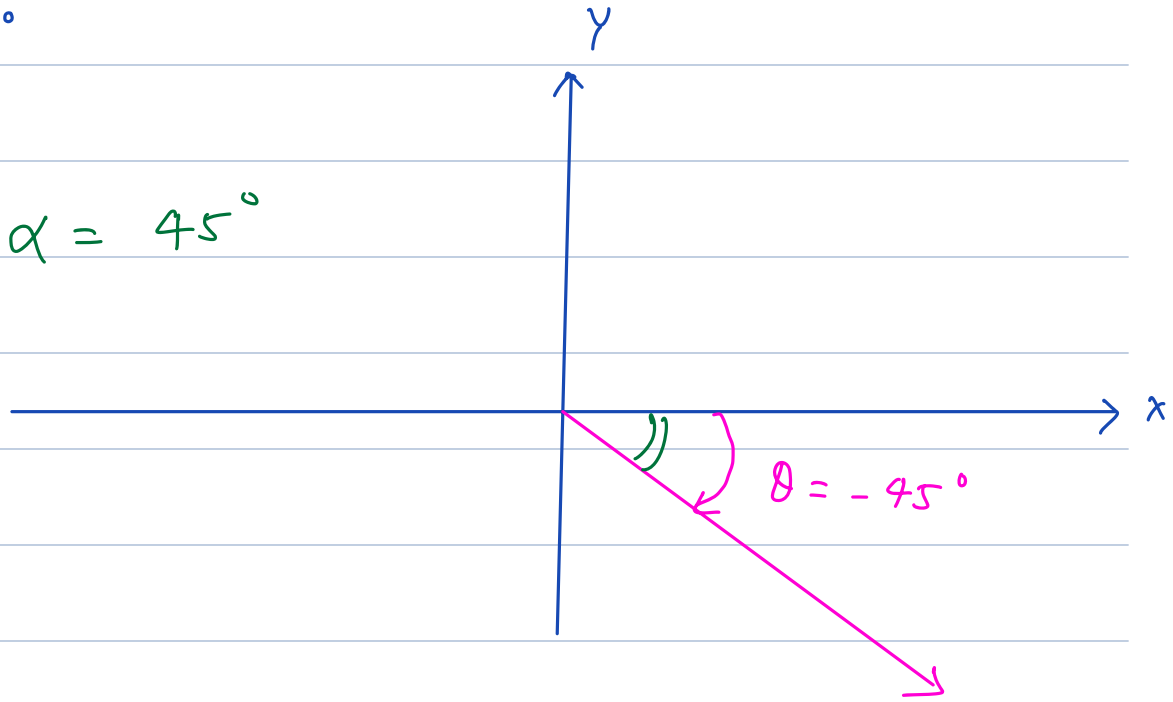


Example:

find α of $\theta = 150^{\circ}$

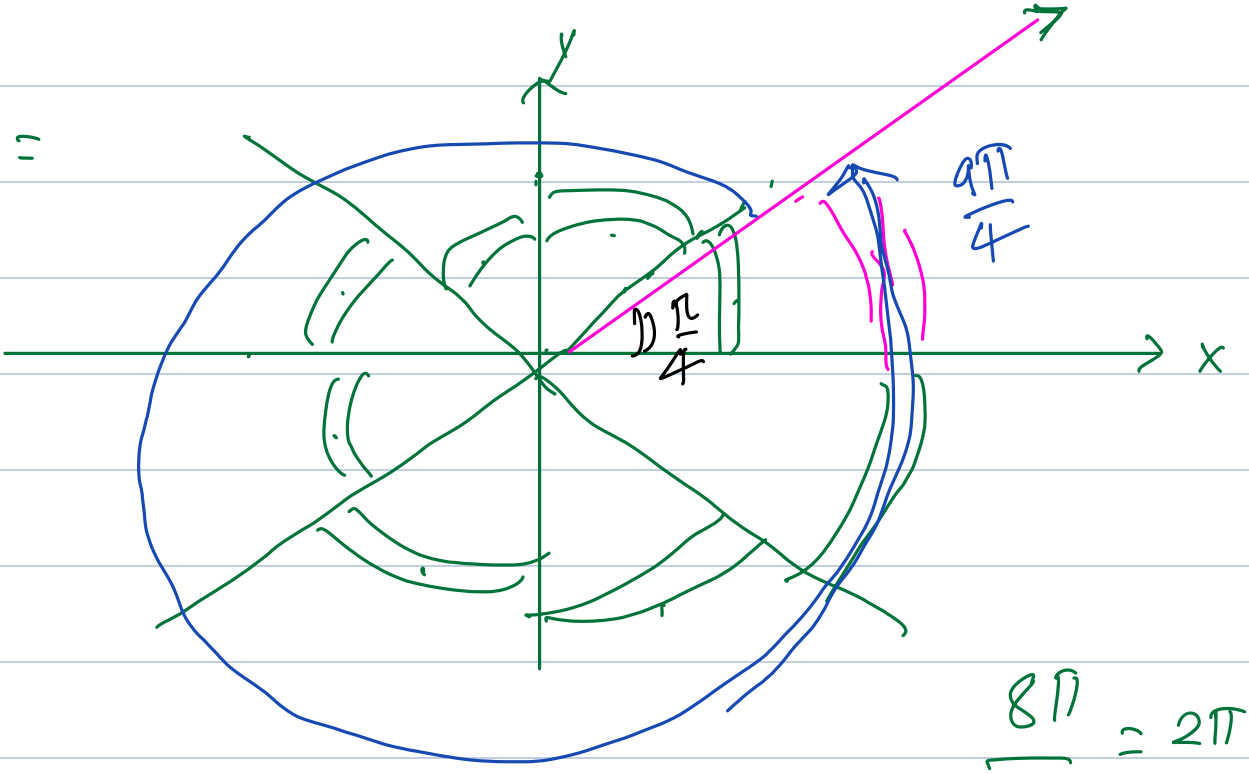


$\theta = -45°$



$\frac{9\pi}{4}$

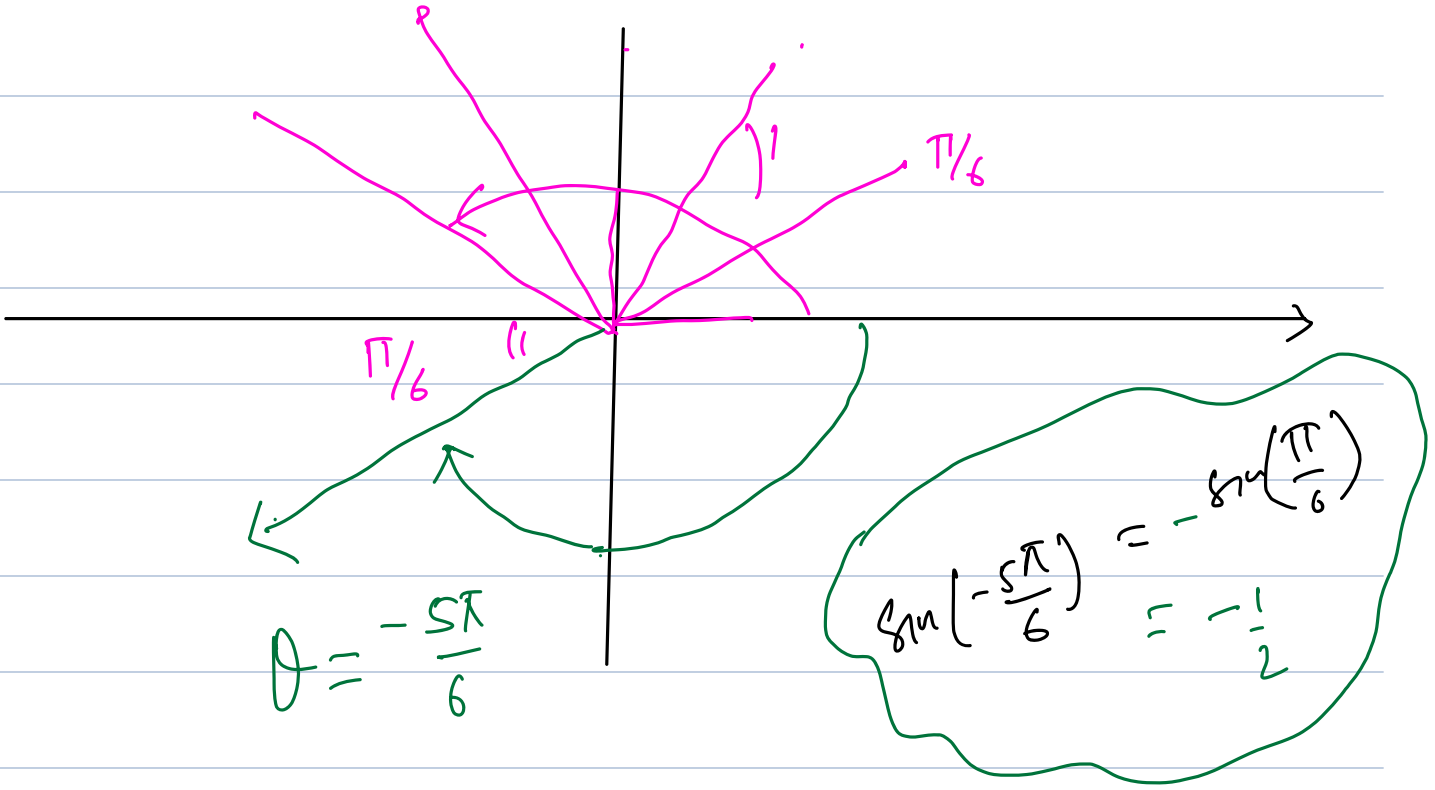
$\frac{9\pi}{4} =$



$\frac{8\pi}{4} = 2\pi$

If $\theta = -\frac{5\pi}{6}$, find α.

$= -5 \cdot \left(\frac{\pi}{6}\right)$



$\theta = -\frac{5\pi}{6}$

$\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$

$= -\frac{1}{2}$

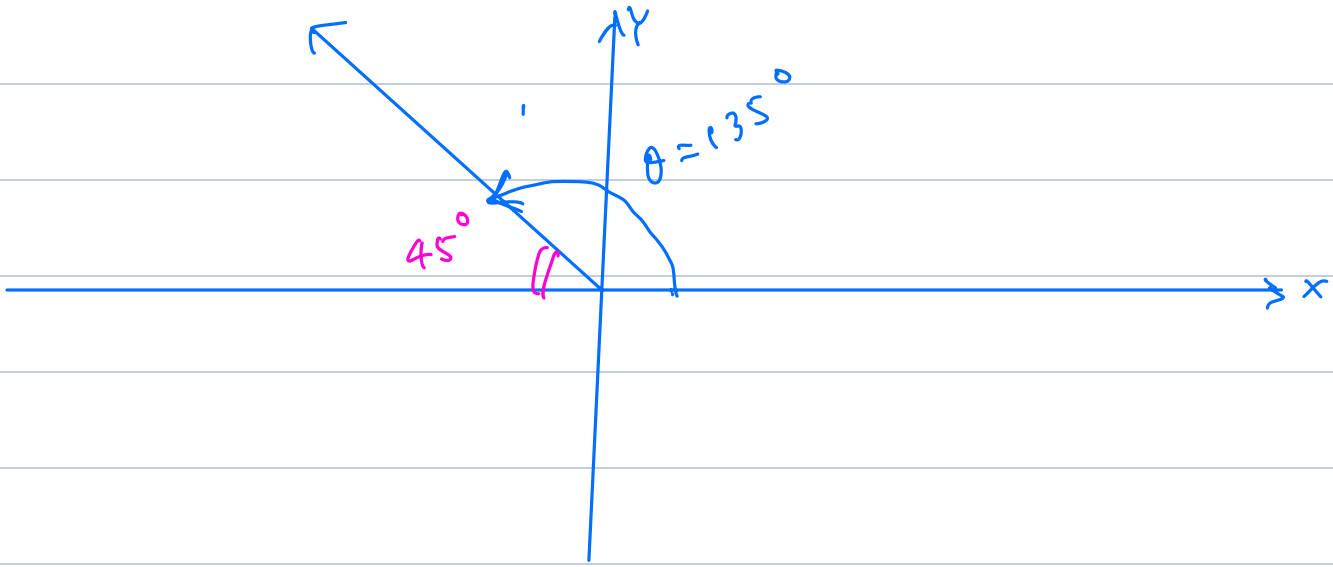
sign

θ α

$\sin\theta$	$=$	$\pm \sin\alpha$
$\cos\theta$	$=$	$\pm \cos\alpha$
$\tan\theta$	$=$	$\pm \tan\alpha$
$\csc\theta$	$=$	$\pm \csc\alpha$
$\sec\theta$	$=$	$\pm \sec\alpha$
$\cot\theta$	$=$	$\pm \cot\alpha$

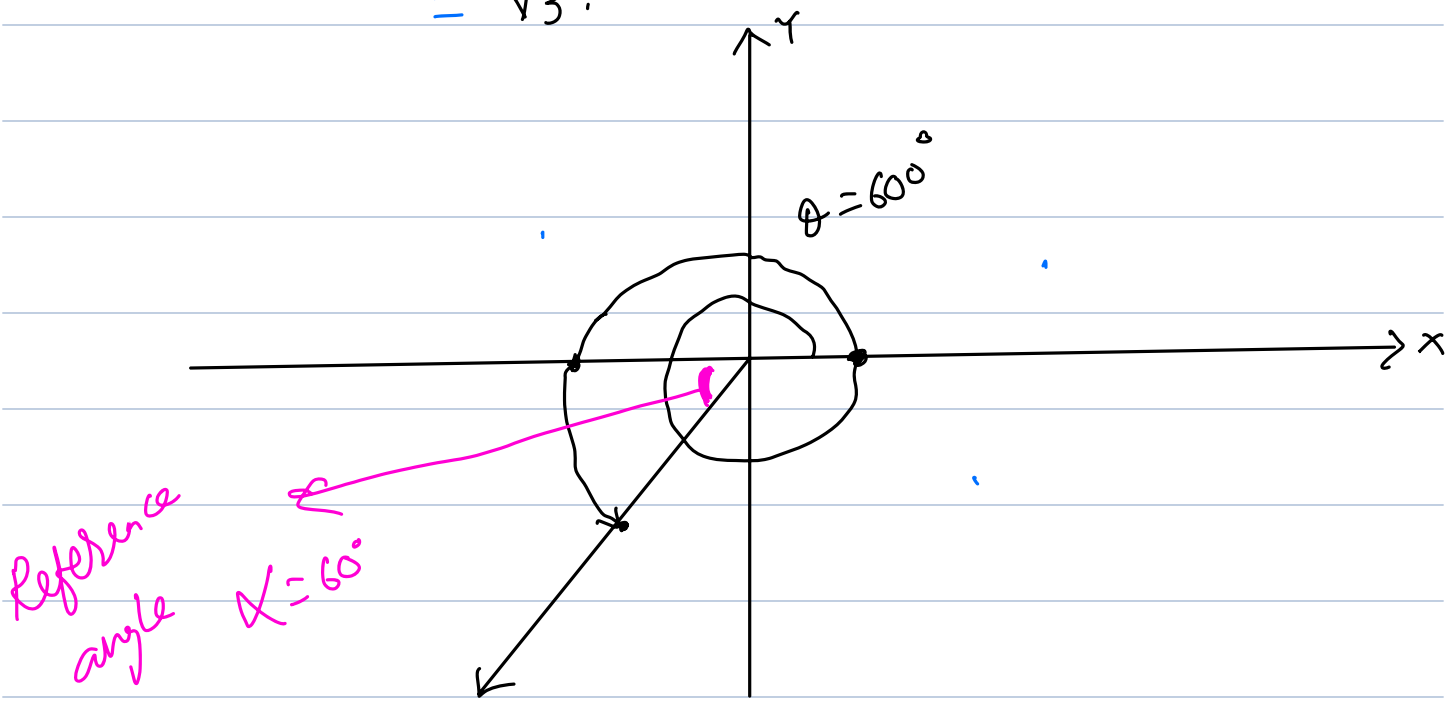
* $\cos 135^\circ = -\cos 45^\circ$

$= -\frac{\sqrt{2}}{2}$



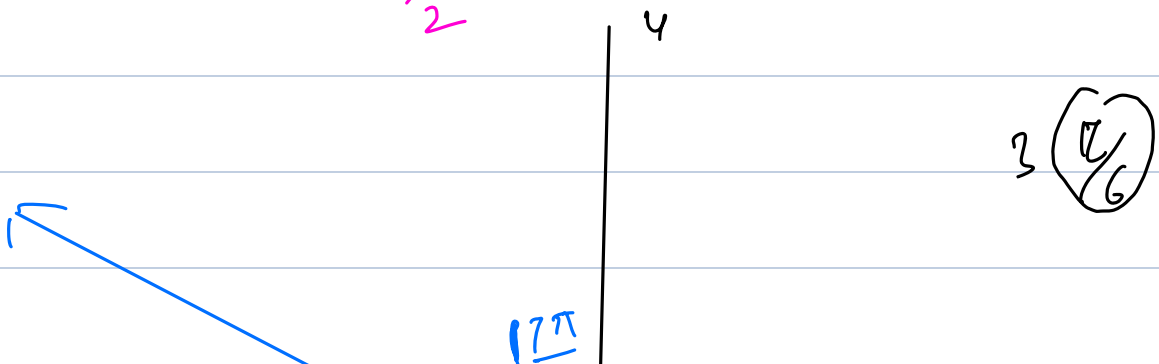
. $\tan 600^\circ = \tan 60^\circ$

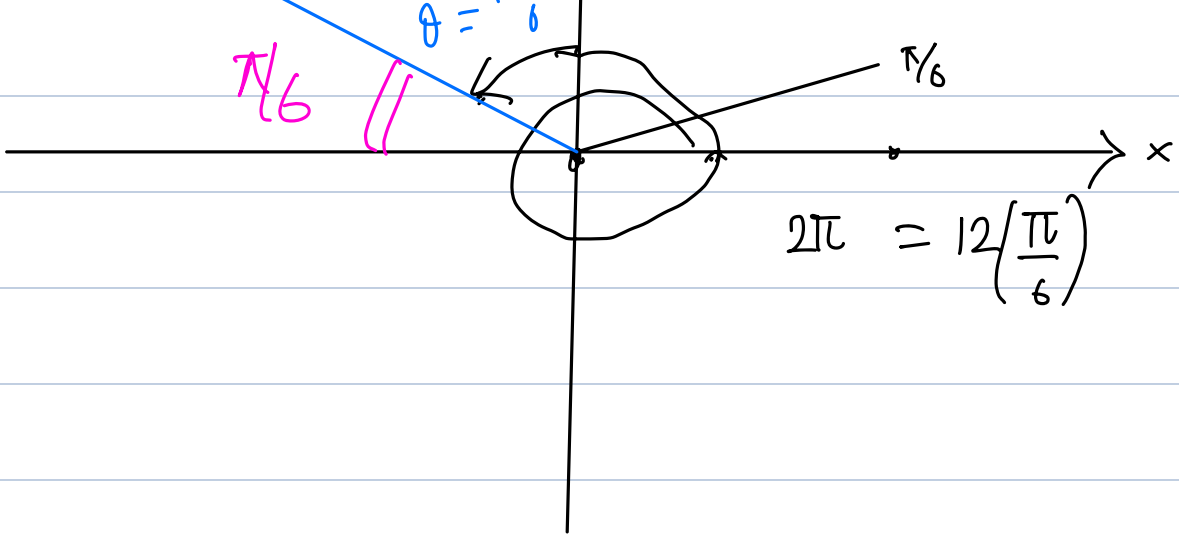
$= \sqrt{3}$.



$\sin\left(\frac{17\pi}{6}\right) = \sin\frac{\pi}{6}$

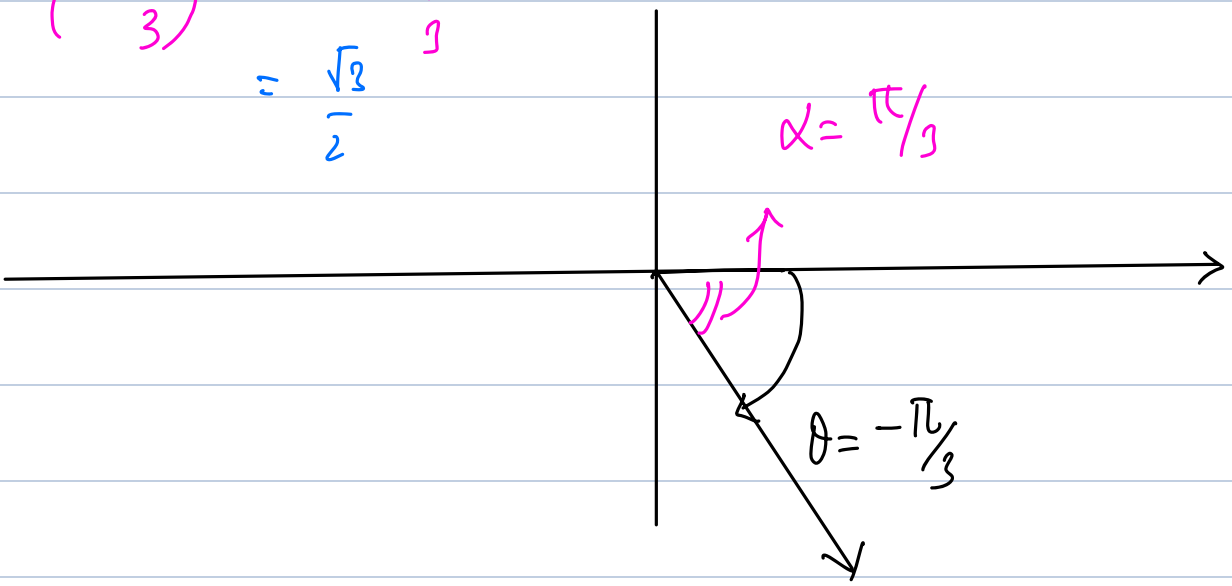
$= \frac{1}{2}$





$$\cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$



* If angle θ is a quadrantal angle, draw the angle, pick a point on its terminal side and apply the definition of trig. function.

$$\sin\theta = \frac{b}{r}, \quad \cos\theta = \frac{a}{r}, \ldots$$

* If the angle θ lies in a quadrant.

1. find the reference angle (α)
2. find the value of trig. function at α.
3. Adjust the sign (+ or −) of the value of the trig. function based on quadrant in which θ lies.

Q: Let $\cos\theta = -\frac{2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$; $\frac{\pi}{2} < \theta < \pi$ II quadrant

Find the rest trig. function.

Solution:

We see $\cos\theta < 0$ in second quadrant.

The angle θ lies in second quadrant.

$\sin\theta$ / $\csc\theta$ are positive.

But: $\cos\theta$, $\tan\theta$, $\sec\theta$, $\cot\theta$ are negative.

Let α be a reference angle.

$\cos\alpha = \frac{2}{\sqrt{5}} = \frac{\text{adj}}{\text{hyp}} = \frac{a}{r}$ $\quad a = 2$, $r = \sqrt{5}$

opp: $\Rightarrow$ $c^2 = a^2 + b^2$

$(\sqrt{5})^2 = 2^2 + b^2$

$5 = 4 + b^2$

$-4 \quad -4 \Rightarrow b^2 = 5 - 4 = 1$

$b = 1$

$\sin\alpha = \frac{b}{r} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\alpha = \frac{b}{a} = \frac{1}{2}$

$\operatorname{cosec}\alpha = \frac{r}{b} = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\sec\alpha = \frac{r}{a} = \frac{\sqrt{5}}{2}$

$\cot\alpha = \frac{a}{b} = \frac{2}{1} = 2$

$\sin\theta = \frac{\sqrt{5}}{5}$, $\operatorname{cosec}\theta = \sqrt{5}$

$\cos\theta = -\frac{2\sqrt{5}}{5}$, $\tan\theta = -\frac{1}{2}$

$\sec\theta = -\frac{\sqrt{5}}{2}$, $\cot\theta = -2$

Q: Let $\tan\theta = -3$ and $\cos\theta < 0$

Find the rest.

→ $\tan\theta = -3 < 0$

$\cos\theta < 0$

θ lies in II quadrant.

$\tan\alpha = \frac{3}{1} = \frac{b}{a}$ $\quad b = 3$, $a = 1$

$r = \sqrt{a^2+b^2}$

$= \sqrt{3^2+1^2}$

$= \sqrt{9+1} = \sqrt{10}$.

$\sin\alpha = \frac{b}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\cos\theta = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

$\sin\theta = \frac{3\sqrt{10}}{10}$