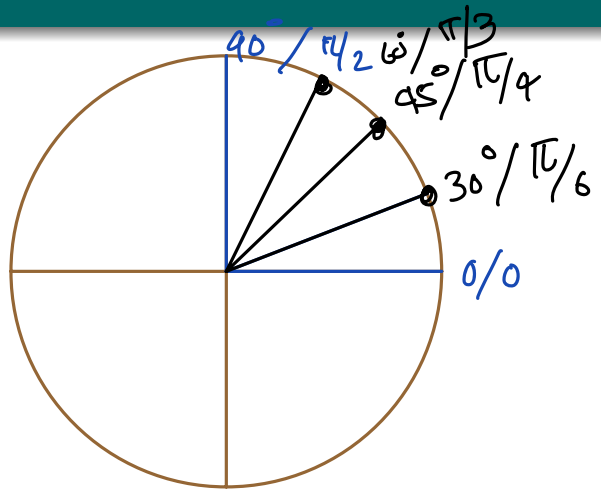
Lecture 2 (Part 2)
Computing the Values of Trigonometric Functions of Acute Angles

Note:

There is a connection between 30° & 60°.

We will start with 45° first.



Objectives

1. Find the Exact Values of the Trigonometric Functions of $\frac{\pi}{4} = 45^\circ$
2. Find the Exact Values of the Trigonometric Functions of $\frac{\pi}{6} = 30^\circ$ and $\frac{\pi}{3} = 60^\circ$
3. Use a Calculator to Approximate the Values of the Trigonometric Functions
4. Solve Applied Problems Involving Right Triangles

Objective 1

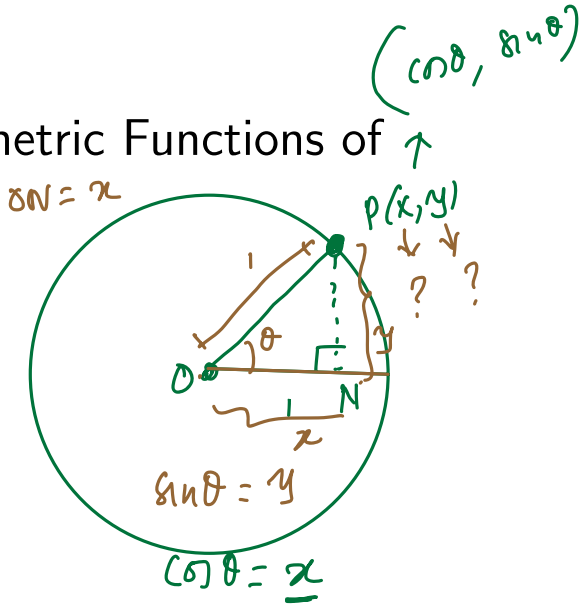
Find the Exact Values of the Trigonometric Functions of

$$\frac{\pi}{4} = 45^\circ$$

$\sin \pi/4$
$\cos \pi/4$
$\tan \pi/4$

$\cot \pi/4$
$\sec \pi/4$
$\csc \pi/4$

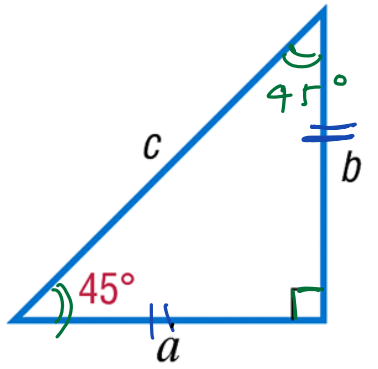
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Example 1: Trigonometric Functions of $\frac{\pi}{4} = 45°$ (1 of 3)

Find the exact values of the six trigonometric functions of $\frac{\pi}{4} = 45°$.

Since one angle of the right triangle is 45°, the other acute angle must also be 45° — so the triangle is isosceles, and sides a and b are equal. Because the value of a trigonometric function depends only on the angle, not on the size of the triangle, we're free to pick any convenient value for $a = b > 0$.



(a)

$a = b$

1

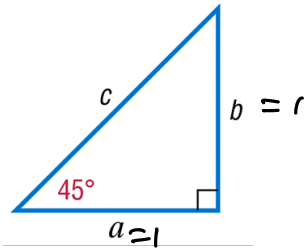
Example 1: Trigonometric Functions of $\frac{\pi}{4} = 45^\circ$ (2 of 3)

For simplicity, take $a = b = 1$.

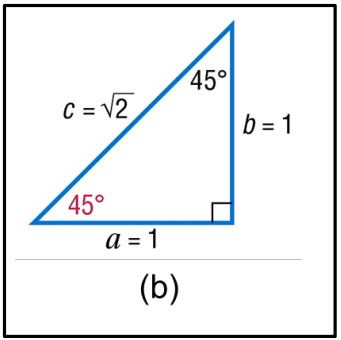
Then, by the Pythagorean Theorem, find c.

$$c^2 = 1^2 + 1^2 = 1 + 1 = 2$$

$$c = \sqrt{2}$$



(a)



(b)

Example 1: Trigonometric Functions of $\frac{\pi}{4} = 45^\circ$ (3 of 3)

Using the ratio definitions, then the Quotient and Reciprocal Identities, find all six functions.

$\sin 45^\circ = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

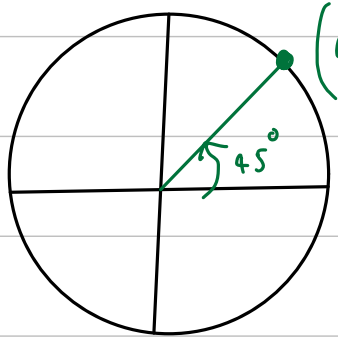
$\cos 45^\circ = \sin \pi/4 = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = \tan \pi/4 = 1$

$\csc 45^\circ = \csc \frac{\pi}{4} = \sqrt{2}$

$\sec 45^\circ = \sec \frac{\pi}{4} = \sqrt{2}$

$\cot 45^\circ = \cot \pi/4 = 1$



$\left(\cos \frac{\pi}{4}, \sin \pi/4\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Example 2: Finding the Exact Value of a Trigonometric Expression

Find the exact value of each expression.

(a) $(\cos 45^\circ)(\cot 45^\circ)$

$= \left(\frac{\sqrt{2}}{2}\right) \cdot 1$

$= \frac{\sqrt{2}}{2}$

(b) $\left(\csc \frac{\pi}{4}\right)\left(\tan \frac{\pi}{4}\right)$

$= (\sqrt{2}) \cdot 1$

$= \sqrt{2}$

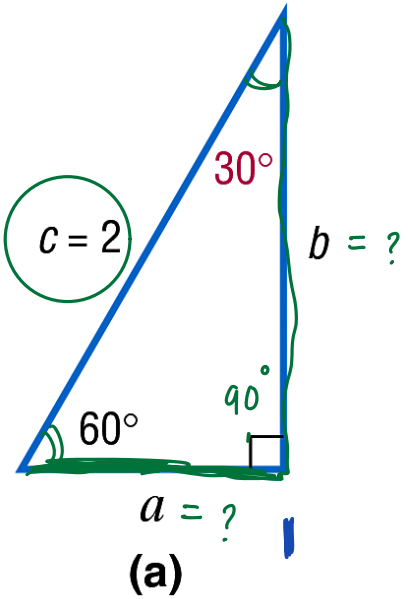
Objective 2

Find the Exact Values of the Trigonometric Functions of $\frac{\pi}{6} = 30^\circ$ and $\frac{\pi}{3} = 60^\circ$

Example 3: Trigonometric Functions of 30° and 60° (1 of 6)

Find the exact values of the six trigonometric functions of $\frac{\pi}{6} = 30°$ and $\frac{\pi}{3} = 60°$.

Build a right triangle with one angle 30°; the third angle is then 60°. Figure (a) shows such a triangle with hypotenuse 2. Our task is to determine a and b.



(a)

congruent triangle

$c^2 = a^2 + b^2$

$\Rightarrow 2^2 = 1^2 + b^2$

$\Rightarrow 4 = 1 + b^2$

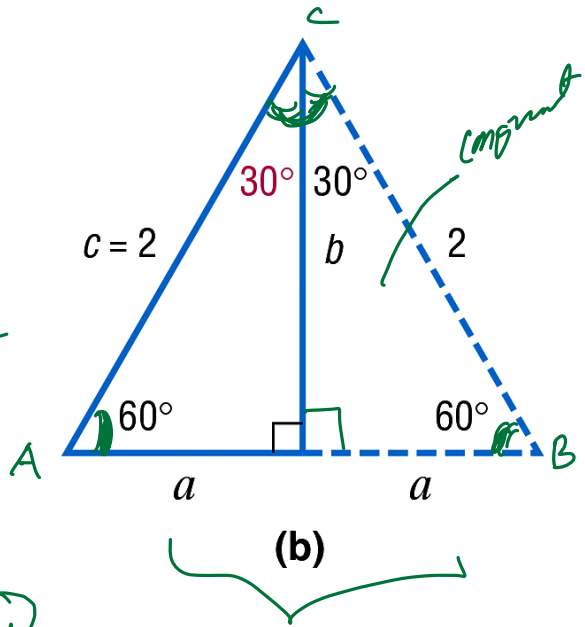
$-1 \quad -1$

$3 = b^2 \Rightarrow b = \sqrt{3}$

Example 3: Trigonometric Functions of 30° and 60° (2 of 6)

Place a second triangle, congruent to the first, next to it as in Figure (b). The combined triangle has three 60° angles, so it's equilateral — every side has length 2. In particular the base is $2a$, so $a =$ _____.

By the Pythagorean Theorem, b satisfies



$\triangle ABC$ is EQUILATERAL

All sides are equal.

from fig: $a + a = 2$

$\Rightarrow 2a = 2 \Rightarrow a = 1$

Example 3: Trigonometric Functions of 30° and 60° (3 of 6)

Using the triangle in Figure (c), and the fact that 30° and 60° are complementary, find $\sin 30°$, $\cos 30°$ and $\sin 60°$.

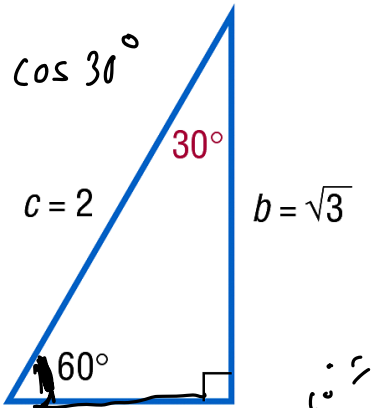
$\sin 60° = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} = \cos(90° - 60°) = \cos 30°$

$\sin 30° = \cos(90° - 30°) = \cos 60°$

$\sin 30° = \frac{1}{2}$ $\quad \cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

$\cot 30° = \tan(90° - 30°) = \tan 60° = \sqrt{3}$



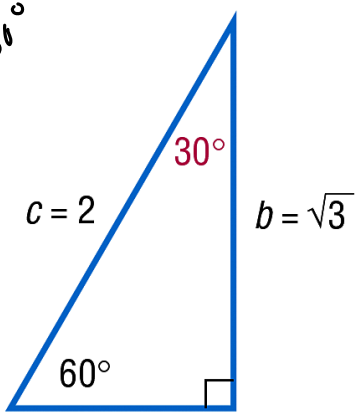
(c)

$\sec 60° = \frac{c}{a} = \frac{2}{1} = 2$

Example 3: Trigonometric Functions of 30° and 60° (4 of 6)

Continue with tan and cot of 30° and 60°.

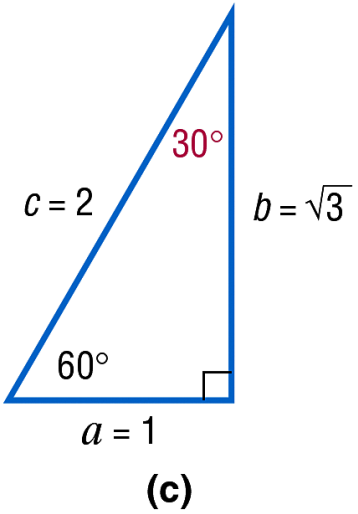
$\sec\frac{\pi}{3} = \sec 60° = \frac{1}{\cos 60°} = \frac{1}{(1/2)} = 2$



(c)

Example 3: Trigonometric Functions of 30° and 60° (5 of 6)

Finish with csc and sec of 30° and 60°.



Example 3: Trigonometric Functions of 30° and 60° (6 of 6)

θ (rad)	θ (deg)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\pi/6$	30°	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$	$\sqrt{3}$
$\pi/4$	45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\pi/3$	60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2\sqrt{3}/3$	2	$\sqrt{3}/3$

Example 4: Finding the Exact Value of a Trigonometric Expression (1 of 2)

Find the exact value of each expression.

(a) $\sin 60^\circ \cos 45^\circ$ (b) $\tan \frac{\pi}{6} + \cos \frac{\pi}{4}$ (c) $\tan^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{6}$

(a) $\sin 60^\circ \cos 45^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4}$

Example 4: Finding the Exact Value of a Trigonometric Expression (2 of 2)

(b) $\tan\frac{\pi}{6} + \cos\frac{\pi}{4}$

$= \frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2}$

$= \frac{2\sqrt{3} + 3\sqrt{2}}{6}$

(c) $\tan^2\frac{\pi}{4} + \sin^2\frac{\pi}{6}$

$= (1)^2 + \left(\frac{1}{2}\right)^2$

$= 1 + \frac{1}{4}$

$= \frac{5}{4}$.

Objective 3

Use a Calculator to Approximate the Values of the Trigonometric Functions of Acute Angles

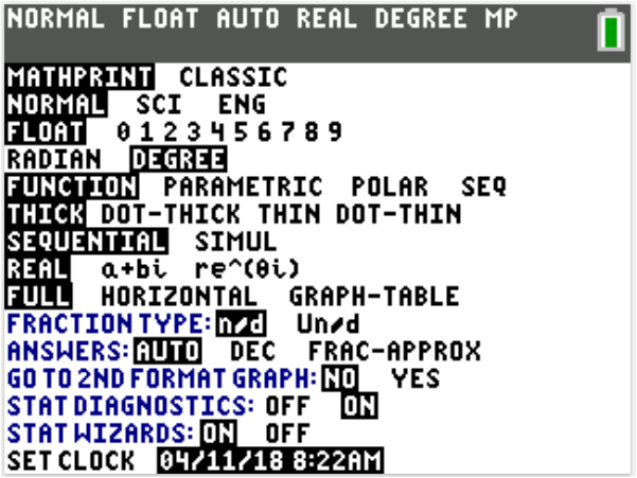
Example 5: Approximating Trigonometric Functions with a Calculator (1 of 4)

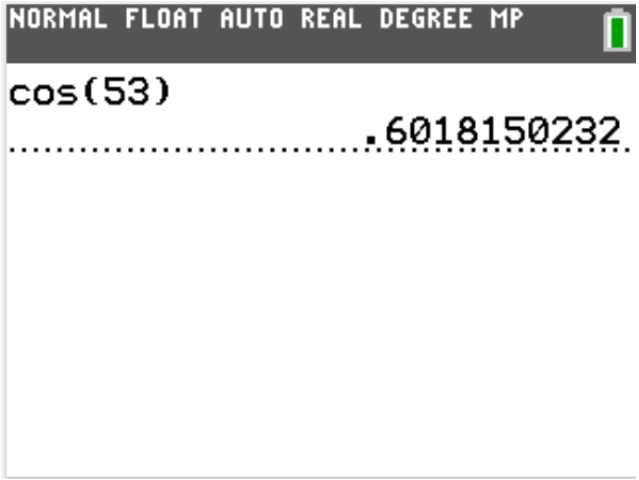
Use a calculator to find the approximate value of each of the following. Express your answer rounded to two decimal places.

(a) $\cos 53^\circ$ (b) $\csc 68^\circ$ (c) $\tan \frac{5\pi}{12}$

Example 5: Approximating Trigonometric Functions with a Calculator (2 of 4)

(a) First, set the calculator's MODE to degrees.



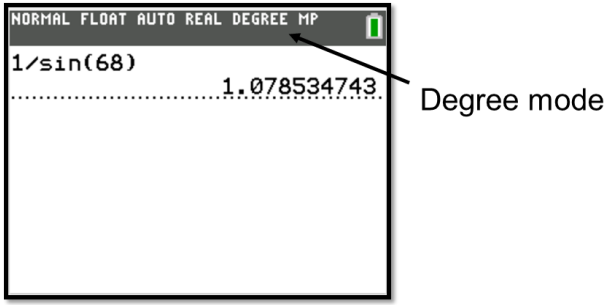


$\cos 53^\circ \approx$ _____

Example 5: Approximating Trigonometric Functions with a Calculator (3 of 4)

(b) Most calculators have no csc key, since the manufacturer assumes you know some trigonometry. To find $\csc 68^\circ$, use the reciprocal identity

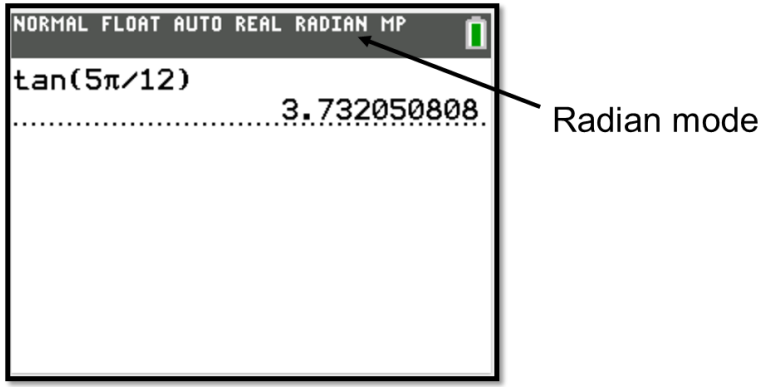
$$\csc 68^\circ = \frac{1}{\sin 68^\circ}$$



$$\csc 68^\circ \approx _____$$

Example 5: Approximating Trigonometric Functions with a Calculator (4 of 4)

(c) Set the MODE to radians.



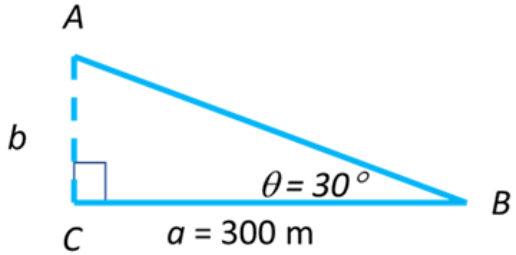
$$\tan\frac{5\pi}{12} \approx _____$$ (rounded to two decimal places)

Objective 4

Solve Applied Problems Involving Right Triangles

Example 6: Finding the Width of a River (1 of 2)

A surveyor measures the width of a river by setting up a transit at point C on one bank and sighting a point A directly across. After turning 90° at C, she walks 300 meters to point B. At B, the angle θ is measured as 30°. What is the width of the river, rounded to the nearest meter?



Example 6: Finding the Width of a River (2 of 2)

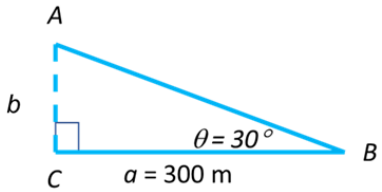
We want the length of side b. We know a and θ; b is opposite θ and a is adjacent to θ.

$$\tan\theta = \frac{b}{a}$$

$$\Rightarrow b = (a)(\tan\theta)$$

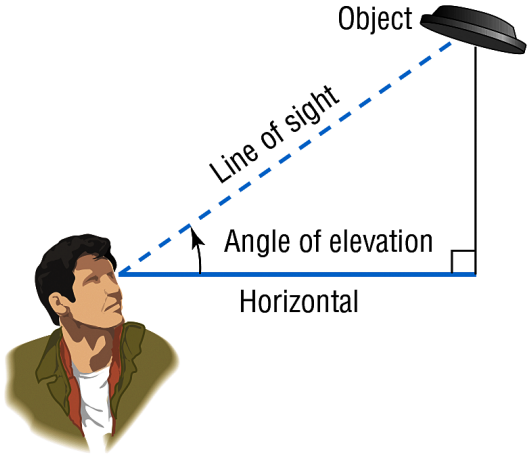
$$= (300)(\tan 30^\circ)$$

$$= 173.2 \text{ meters}$$



Angle of Elevation

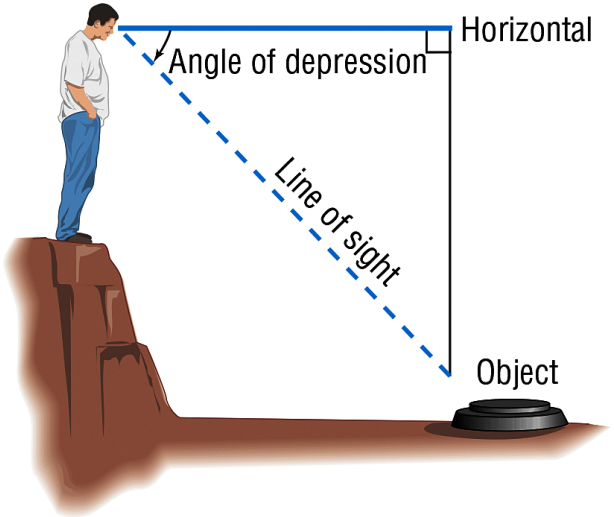
Vertical heights are often measured using an angle of elevation or an angle of depression. When looking up at an object, the acute angle between the horizontal and the line of sight is the **angle of elevation**.



(a)

Angle of Depression

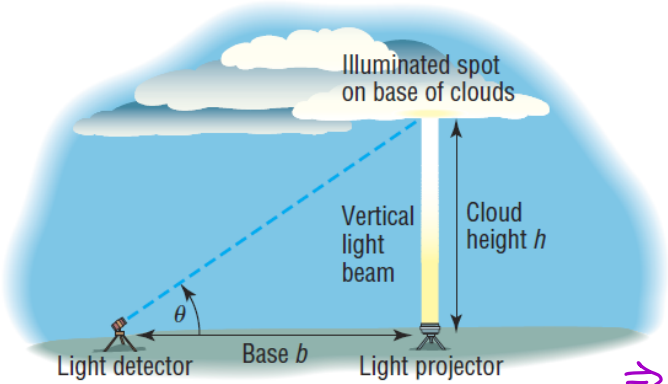
When looking down at an object, the acute angle between the line of sight and the horizontal is the **angle of depression**.



(b)

Example 7: Finding the Height of a Cloud (1 of 2)

Meteorologists find cloud height with a ceilometer: a light projector aims a vertical beam at the cloud base, and a light detector some distance away scans for it. At Midway Airport, the detector sat 300 feet from the projector, and the angle of elevation from the detector to the base of the cloud was 75°. What was the height of the cloud cover?



$$\tan\theta = \frac{h}{b}$$

$$\tan 75^\circ = \frac{h}{300}$$

$$\Rightarrow h = (300)\tan 75^\circ$$

Example 7: Finding the Height of a Cloud (2 of 2)

To find the height h, use the fact that h is opposite the 75° angle and 300 feet is adjacent to it.

$$\tan 75^\circ = \frac{h}{300}$$

$$\Rightarrow \quad h = (\tan 75^\circ)(300)$$

$$= 1120 \text{ feet}$$

