Lecture 2
Right Triangle Trigonometry

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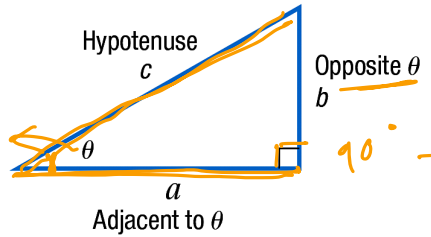
Objectives

1. Find the Values of Trigonometric Functions of Acute Angles
2. Use Fundamental Identities
3. Find the Values of the Remaining Trigonometric Functions, Given the Value of One of Them
4. Use the Complementary Angle Theorem

Objective 1

Find the Values of Trigonometric Functions of Acute Angles

Definition (1 of 2)



acute

c = Hypotenuse
b = opposite
a = adjacent

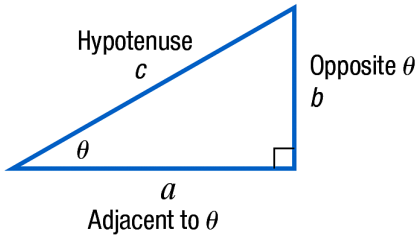
90° → $\frac{\pi}{2}$ radian

For a right triangle with an acute angle θ, the six ratios formed from its three side lengths are called the **trigonometric functions of** θ:

Function	Abbreviation	Value
sine of θ	$\sin\theta$	
cosine of θ	$\cos\theta$	
tangent of θ	$\tan\theta$	
cosecant of θ	$\csc\theta$	
secant of θ	$\sec\theta$	
cotangent of θ	$\cot\theta$	

cosec θ

Definition (2 of 2)



Reciprocal: If $a \neq 0$ then reciprocal of a is $\frac{1}{a}$.

- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{b}{c}$
- $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{a}{c}$
- $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{b}{a}$
- $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{c}{b}$
- $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{c}{a}$
- $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{a}{b}$

Example 1: Finding the Values of Trigonometric Functions (1 of 3)

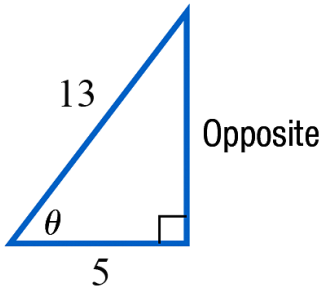
Find the value of each of the six trigonometric functions of the angle θ.

Given:

hypotenuse $(c) = 13$

Adjacent $(a) = 5$

$b = ?$



$$c^2 = a^2 + b^2$$

Example 1: Finding the Values of Trigonometric Functions (2 of 3)

The two given sides tell us the hypotenuse and the side adjacent to θ. To find the length of the missing (opposite) side, apply the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$

$$-a^2 \quad -a^2$$

$$c^2 - a^2 = b^2$$

$$\Rightarrow 13^2 - 5^2 = b^2$$

$$\Rightarrow 169 - 25 = b^2$$

$$\Rightarrow 144 = b^2 \Rightarrow b = \pm\sqrt{144} = \pm 12$$

We take positive b: $\boxed{b = 12}$

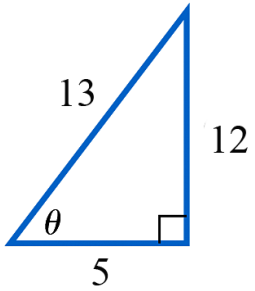
Example 1: Finding the Values of Trigonometric Functions (3 of 3)

Now that all three side lengths are known, use the ratio definitions to find each of the six trigonometric functions.

$\sin\theta = \frac{b}{c} = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\cos\theta = \frac{a}{c} = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

$\tan\theta = \frac{b}{a} = \frac{12}{5}$ $\quad \cot\theta = \frac{5}{12}$



$$\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{12}{13}\right)}{\left(\frac{5}{13}\right)} = \frac{12}{13} \cdot \frac{13}{5} = \frac{12}{5} = \tan\theta$$

Objective 2

Use Fundamental Identities

Identities (1 of 2)

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$\Rightarrow \sin\theta \cdot \csc\theta = 1$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta}$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Example 2: Finding the Remaining Functions, Given $\sin\theta$ and $\cos\theta$ (1 of 2)

Given $\sin\theta = \dfrac{\sqrt{10}}{10}$ and $\cos\theta = \dfrac{3\sqrt{10}}{10}$, find the value of each of the four remaining trigonometric functions of θ.

Start from the quotient identity for $\tan\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{\sqrt{10}}{10}\right)}{\left(\frac{3\sqrt{10}}{10}\right)} = \frac{\sqrt{10}}{10} \cdot \frac{10}{3\sqrt{10}} = \frac{1}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{10}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{10\sqrt{10}}{(\sqrt{10})^2} = \frac{10\sqrt{10}}{10} = \sqrt{10}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{10}{3\sqrt{10}} = \frac{10 \cdot \sqrt{10}}{3\sqrt{10} \cdot \sqrt{10}} = \frac{10\sqrt{10}}{3(10)} = \frac{\sqrt{10}}{3}, \quad \cot\theta = \frac{1}{\tan\theta} = \frac{3}{1} = 3.$$

Example 2: Finding the Remaining Functions, Given $\sin\theta$ and $\cos\theta$ (2 of 2)

Then use the reciprocal identities to find $\csc\theta$, $\sec\theta$, and $\cot\theta$.

$* \quad \sin\theta = \frac{b}{h} \qquad \cos\theta = \frac{a}{h}$

$\sin^2\theta = \frac{b^2}{h^2} \qquad (\cos\theta)^2 = \cos^2\theta = \frac{a^2}{h^2}$

$\sin^2\theta + \cos^2\theta = \frac{b^2}{h^2} + \frac{a^2}{h^2} = \frac{b^2+a^2}{h^2} = \frac{h^2}{h^2} = 1$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

- $\tan^2\theta + 1 = \sec^2\theta$
- $\cot^2\theta + 1 = \csc^2\theta$

$\tan^2\theta + 1 = \frac{b^2}{a^2} + 1 = \frac{b^2}{a^2} + \frac{a^2}{a^2} = \frac{b^2+a^2}{a^2} = \frac{h^2}{a^2} = \left(\frac{h}{a}\right)^2 = \left(\frac{c}{a}\right)^2 =$

$\Rightarrow \cot^2\theta + 1 = \csc^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Fundamental Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Example 3: Using Identities to Find an Exact Value

Find the exact value of each expression. Do not use a calculator.

(a) $\dfrac{\sin 39^\circ}{\cos 39^\circ} - \tan 39^\circ$

(b) $\cos^2 \dfrac{\pi}{4} + \dfrac{1}{\csc^2 \frac{\pi}{4}}$

$\tan 39^\circ - \tan 39^\circ$

$= 0$

$= \cos^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{4}$

$= 1$

Objective 3

Find the Values of the Remaining Trigonometric Functions, Given the Value of One of Them

Example 4: Given $\sin\theta$, θ Acute (1 of 5)

Given that $\sin\theta = \frac{1}{2}$ and θ is an acute angle, find the exact value of each of the remaining five trigonometric functions of θ.

We will solve this two ways: first from the definition of the trigonometric functions, then again using fundamental identities.

- Given: $\sin\theta = \frac{1}{2}$, $\cos\theta =$

$\tan\theta =$

$\sec\theta =$

$\csc\theta =$

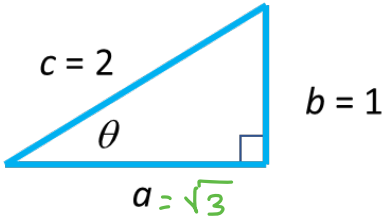
$\cot\theta =$

Example 4: Given $\sin\theta$, θ Acute (2 of 5)

Option 1

Since $\sin\theta = \frac{1}{2} = \frac{b}{c}$, draw a right triangle with acute angle θ, opposite side $b = 1$, and hypotenuse $c = 2$.

The adjacent side a can be found with the Pythagorean Theorem.

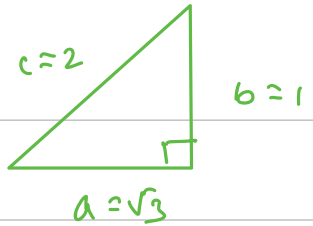


$$c^2 = a^2 + b^2$$
$$\Rightarrow 2^2 = a^2 + 1^2$$
$$\Rightarrow 4 = a^2 + 1$$
$$-1 \qquad -1$$
$$3 = a^2$$
$$\Rightarrow a = \pm\sqrt{3}$$
$$a = \sqrt{3}$$

[Since length is not negative]

Example 4: Given $\sin\theta$, θ Acute (3 of 5)

Using $a^2 + b^2 = c^2$ with $b = 1$ and $c = 2$, solve for a, then use a, b, and c to find the remaining five functions.



$\cos\theta = \sqrt{3}/2$

$\tan\theta = \frac{b}{a} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sec\theta = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc\theta = \frac{2}{1} = 2$

$\cot\theta = \sqrt{3}$

Example 4: Given $\sin\theta$, θ Acute (4 of 5)

Option 2

Begin by finding $\cos\theta$ using the Pythagorean Identity

$$\sin^2\theta + \cos^2\theta = 1 \qquad \text{Equation (5)}$$

$\Rightarrow$ We are given: $\sin\theta = \frac{1}{2}$

We know:

$\sin^2\theta + \cos^2\theta = 1$

$-\sin^2\theta \qquad -\sin^2\theta$

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{1}{2}\right)^2 = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}$

$\cos\theta = \pm\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Example 4: Given $\sin\theta$, θ Acute (5 of 5)

Now use $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$ together with the fundamental identities to get the remaining four functions.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

- $\sec\theta = \frac{1}{\cos\theta}$ [Reciprocal identity]

$$= \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

- $\csc\theta = \frac{1}{\sin\theta} = 2$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Finding the Remaining Functions When One Is Known (1 of 2)

Given the value of one trigonometric function of an acute angle θ, the exact value of each remaining function can be found either of two ways.

Option 1: Using the Definition

1. Draw a right triangle showing the acute angle θ.
2. Label two of the sides using the value of the given function.
3. Find the third side with the Pythagorean Theorem.
4. Use the ratio definitions to find each remaining function.

Finding the Remaining Functions When One Is Known (2 of 2)

Option 2: Using Identities

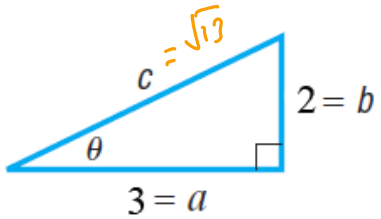
Choose appropriate fundamental identities (reciprocal, quotient, or Pythagorean) and apply them in sequence to reach each remaining function.

Example 5: Given One Function Value, θ Acute (1 of 5)

Given $\tan\theta = \frac{2}{3}$, θ an acute angle, find the exact value of each of the remaining five trigonometric functions of θ.

Solution — Option 1

With $b = 2$ and $a = 3$, find the hypotenuse c using the Pythagorean Theorem.



$c^2 = a^2 + b^2$

$\Rightarrow c = \sqrt{a^2+b^2} = \sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13}$

Example 5: Given One Function Value, θ Acute (2 of 5)

Now apply the ratio definitions using $a = 3$, $b = 2$, and the hypotenuse c found above.

$$\sin\theta = \frac{b}{c} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{a}{c} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\csc\theta = \frac{c}{b} = \frac{\sqrt{13}}{2}$$

$$\sec\theta = \frac{c}{a} = \frac{\sqrt{13}}{3}$$

$$\cot\theta = \frac{a}{b} = \frac{3}{2}$$

Example 5: Given One Function Value, θ Acute (3 of 5)

Solution — Option 2

Since $\tan\theta$ is known, use the Pythagorean Identity that involves $\tan\theta$.

$= \left(\frac{2}{3}\right)$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \left(\frac{2}{3}\right)^2 = \sec^2\theta$$

$$1 + \frac{4}{9} = \sec^2\theta$$

$$\sec\theta = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$$

Now, we have $\tan\theta = \frac{2}{3}$ & $\sec\theta = \frac{\sqrt{13}}{3}$

Example 5: Given One Function Value, θ Acute (4 of 5)

We now know $\tan\theta$ and $\sec\theta$. Use the reciprocal identities to get $\cos\theta$ and $\cot\theta$.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{2}$$

Reciprocal identities

Example 5: Given One Function Value, θ Acute (5 of 5)

To find $\sin\theta$, use the quotient identity relating $\sin\theta$, $\cos\theta$, and $\tan\theta$; then find $\csc\theta$.

$\star$ We know: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\theta \cdot (\cos\theta) = \dfrac{\sin\theta}{\cos\theta} \cdot (\cos\theta)$

$\Rightarrow \sin\theta = \tan\theta \cdot \cos\theta$

$= \dfrac{2}{3} \cdot \dfrac{3\sqrt{13}}{13} = \dfrac{2\sqrt{13}}{13}$

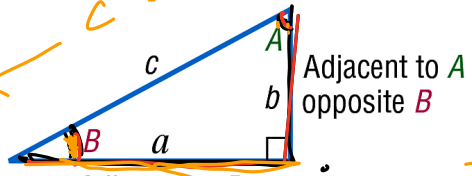
And

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{13}}{2}$

Objective 4

Use the Complementary Angle Theorem

Identities (2 of 2)



$$\sin B = \frac{b}{c} = \cos A \qquad \cos B = \frac{a}{c} = \sin A \qquad \tan B = \frac{b}{a} = \cot A$$

$$\csc B = \frac{c}{b} = \sec A \qquad \sec B = \frac{c}{a} = \csc A \qquad \cot B = \frac{a}{b} = \tan A$$

Theorem

Complementary Angle Theorem

Cofunctions of complementary angles are equal.

Complementary Angles

θ **in degrees**	θ **in radians**
$\sin\theta = \cos(90^\circ - \theta)$	$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$
$\cos\theta = \sin(90^\circ - \theta)$	$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$
$\tan\theta = \cot(90^\circ - \theta)$	$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$
$\csc\theta = \sec(90^\circ - \theta)$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$
$\sec\theta = \csc(90^\circ - \theta)$	$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$
$\cot\theta = \tan(90^\circ - \theta)$	$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

Example 6: Using the Complementary Angle Theorem (1 of 2)

Rewrite each using the cofunction of the complementary angle.

(a) $\sin 79^\circ$ (b) $\tan \frac{\pi}{18}$ (c) $\cos \frac{\pi}{3}$ (d) $\csc \frac{5\pi}{12}$

$\cos(90^\circ - 79^\circ)$

$= \cos(11^\circ)$

$\cot\left(\frac{9\cdot\pi}{9\cdot 2} - \frac{\pi}{18}\right)$

$= \cot\left(\frac{9\pi}{18} - \frac{\pi}{18}\right)$

$= \cot\left(\frac{8\pi}{18}\right)$.

$= \cot\left(\frac{4\pi}{9}\right)$ ✓

$9\cdot\frac{\pi}{9\cdot 2} - \frac{\pi}{18}$

$= \frac{9\pi}{18} - \frac{\pi}{18}$

$\Rightarrow \sec\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)$

$= \sec\left(\frac{6\cdot\pi}{6\cdot 2} - \frac{5\pi}{12}\right)$

$= \sec\left(\frac{\pi}{12}\right)$

Example 6: Using the Complementary Angle Theorem (2 of 2)

Find the exact value of each expression. Do not use a calculator.

(a) $\sec 11^\circ - \csc 79^\circ$ (b) $\dfrac{\sin 37^\circ}{\cos 53^\circ}$

Caveat !!

a) $\sec 11^\circ - \csc 79^\circ$

$= \sec 11^\circ - \sec(90^\circ - 79^\circ)$ ⇒ TRICK.

$= \sec 11^\circ - \sec 11^\circ$

$= 0$

↓ Can we apply Quotient identity ??

[NO]

$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\dfrac{\sin 37^\circ}{\sin(90^\circ - 53^\circ)}$

$= \dfrac{\sin 37^\circ}{\sin 37^\circ} = 1$