Lecture 1
Angles and Their Measure

Pitambar Acharya

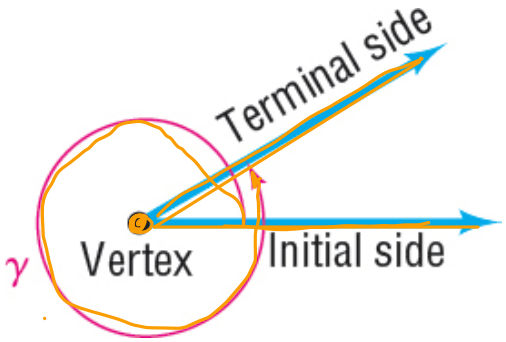
August 24, 2026

Objectives

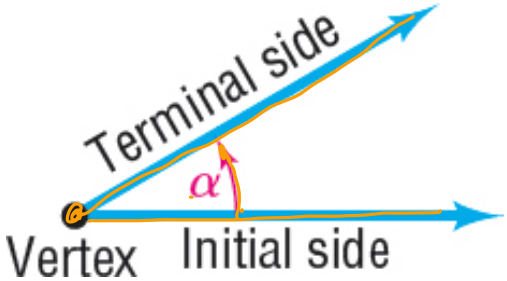
1. Draw Positive and Negative Angles
2. Convert from Degrees to Radians and from Radians to Degrees
3. Find the Length of an Arc of a Circle
4. Find the Area of a Sector of a Circle

Drawing an Angle

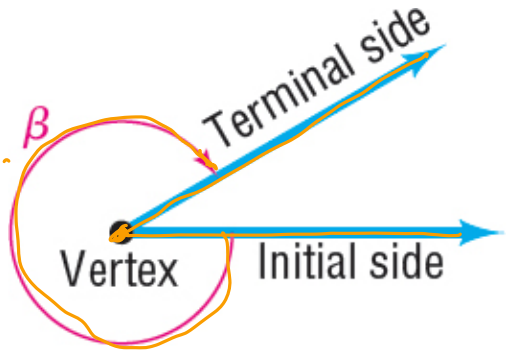
Two rays that share a common endpoint form an **angle**. That shared endpoint is the **vertex**. One ray stays fixed and is called the **initial side**; the ray that rotates away from it is the **terminal side**.



Counterclockwise rotation
Positive angle



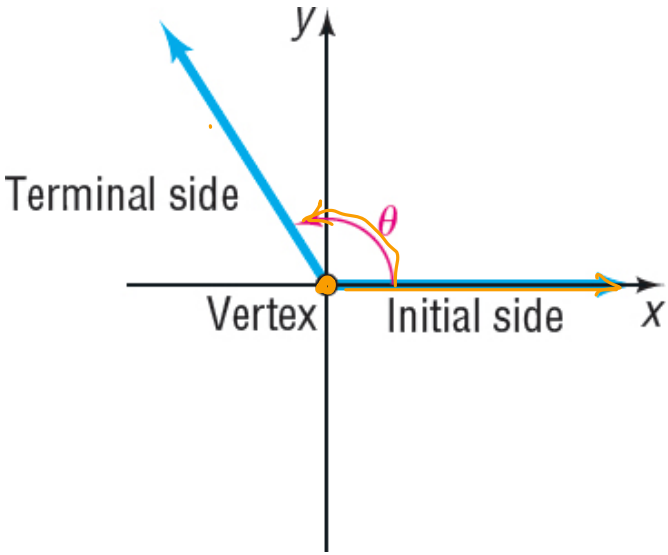
Counterclockwise rotation
Positive angle



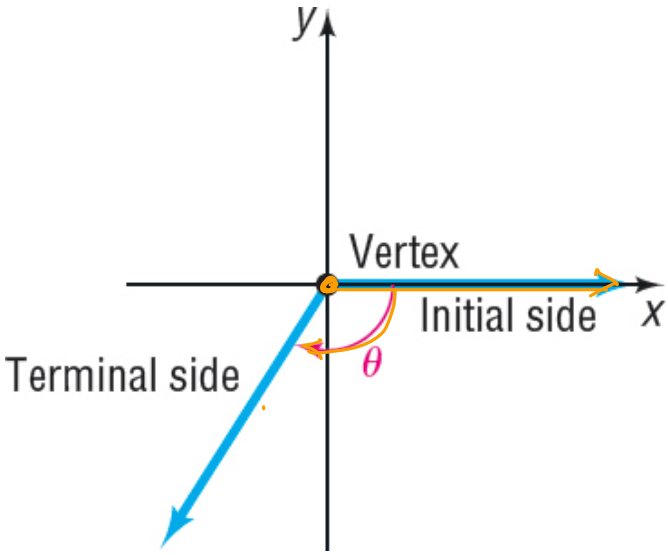
Clockwise rotation
Negative angle

An Angle in Standard Position

We place an angle θ in **standard position** by putting its vertex at the origin and letting its initial side run along the positive x-axis.



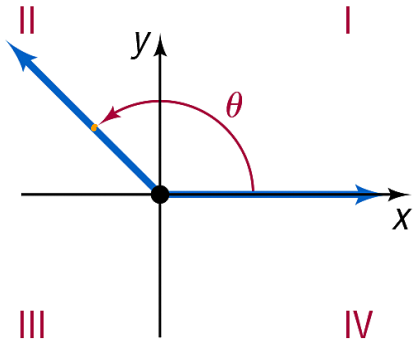
θ is in standard position; θ is positive



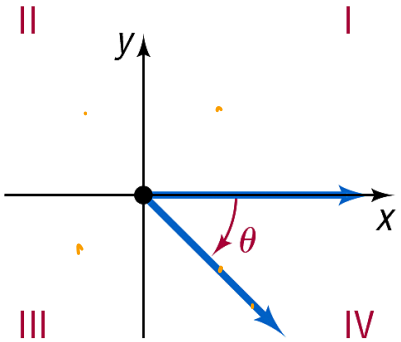
θ is in standard position; θ is negative

Quadrants and Quadrantal Angles

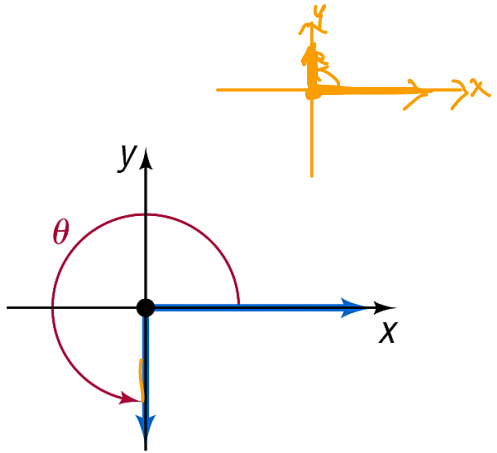
Once θ is in standard position, its terminal side lands in one of two places: inside one of the four quadrants — then we say θ **lies in** that quadrant — or exactly on the x-axis or y-axis, in which case θ is called a **quadrantal angle**.



(a) θ lies in quadrant II



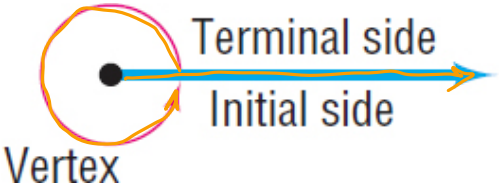
(b) θ lies in quadrant IV



(c) θ is a quadrantal angle

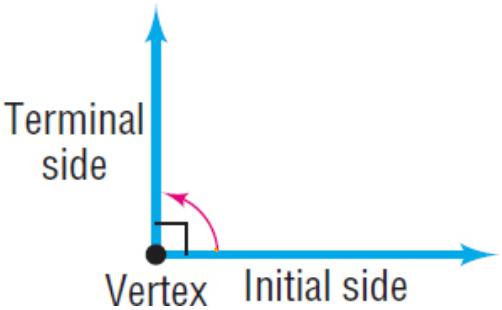
Degrees

Turning the initial side counterclockwise through one complete revolution — until it lands back on itself — sweeps out an angle we call 360 **degrees**, written 360°. A single degree, 1°, is therefore $\frac{1}{360}$ of a full revolution. A **right angle** is a quarter turn, 90°; a **straight angle** is a half turn, 180°.



(a) 1 revolution counterclockwise, 360°

$1^\circ = \frac{1}{360}$ of full revolution



(b) right angle, $\frac{1}{4}$ revolution ccw, 90°

180°



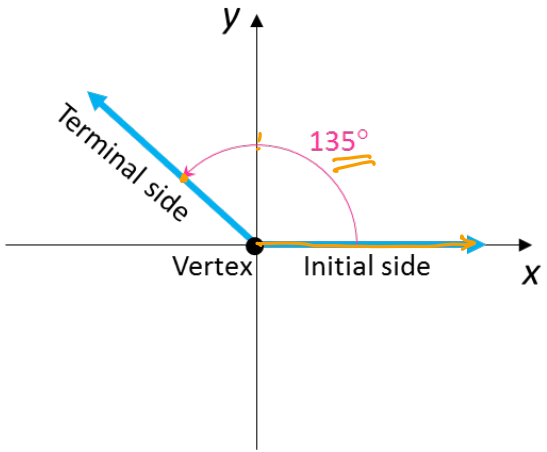
(c) straight angle, $\frac{1}{2}$ revolution ccw, 180°

Example 1: Drawing an Angle (1 of 4)

Draw each angle in standard position.

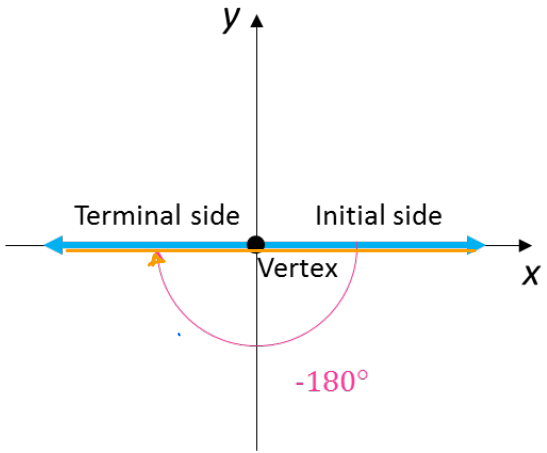
(a) 135° (b) -180° (c) 90° (d) 495°

(a) 135°



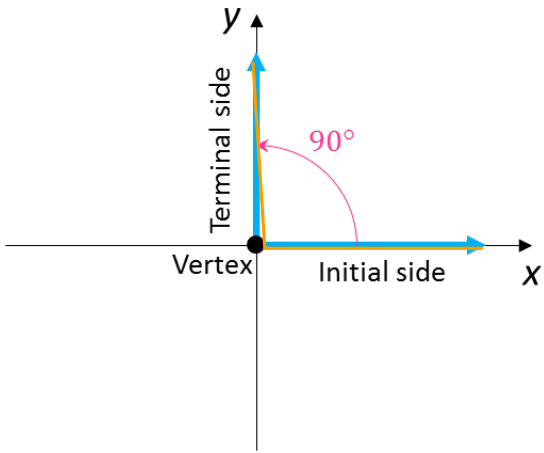
Example 1: Drawing an Angle (2 of 4)

(b) -180°



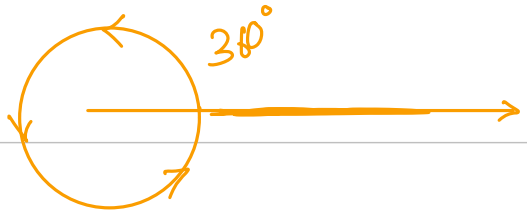
Example 1: Drawing an Angle (3 of 4)

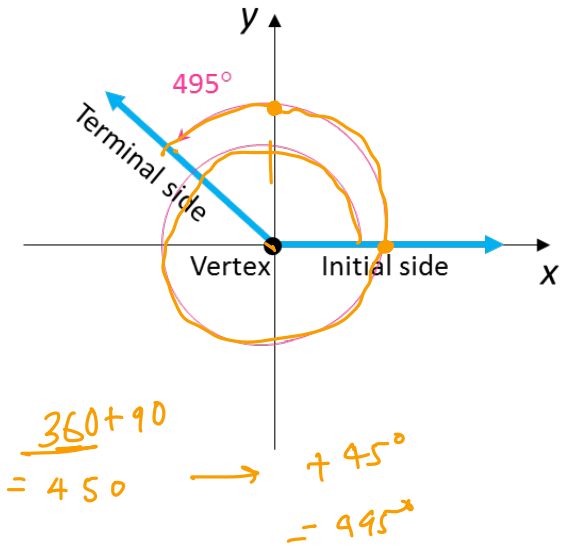
(c) 90°



Example 1: Drawing an Angle (4 of 4)

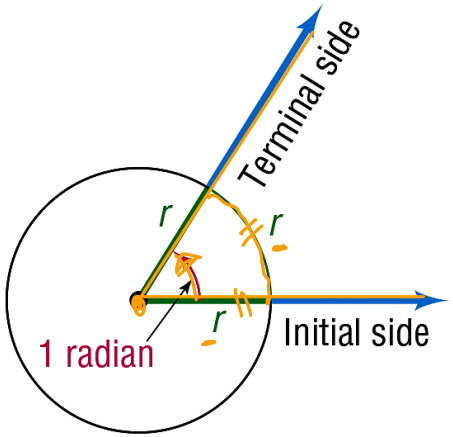
(d) 495°





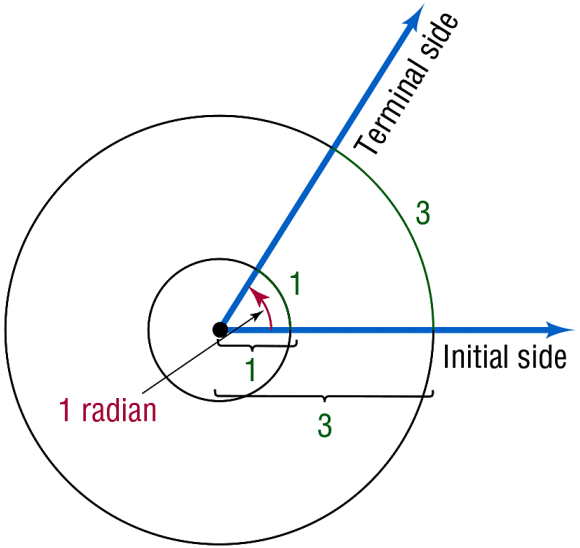
Radians (1 of 2)

A **central angle** has its vertex at the center of a circle, and its two rays cut out (subtend) an arc on that circle. When that arc has the same length as the radius r, we say the central angle measures exactly 1 **radian**.



Radians (2 of 2)

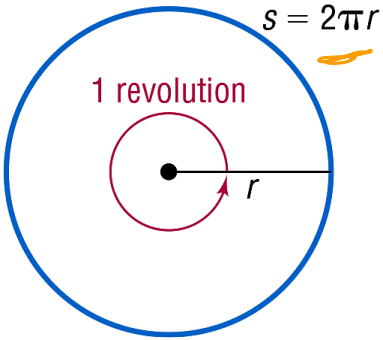
This scales with the circle itself: on a circle of radius 1, a 1-radian central angle subtends an arc of length 1; on a larger circle of radius 3, that same 1-radian angle subtends a longer arc, of length 3.



Objective 2

Convert from Degrees to Radians and
from Radians to Degrees

Properties



1 revolution = 2π radians

$180^\circ = \pi$ radians

1 degree = $\frac{\pi \text{ radians}}{180}$

1 radian = $\frac{180 \text{ degrees}}{\pi}$

Example 2: Converting from Degrees to Radians (1 of 2)

Convert each angle in degrees to radians.

(a) 45° (b) 210° (c) -60° (d) 100°

a) $45^\circ = 45 \cdot 1^\circ$

$= 45 \cdot \frac{\pi}{180}$ radian

$= \frac{\pi}{4}$ radian

b) $210^\circ = 210 \cdot 1^\circ$

$= 210 \cdot \frac{\pi}{180}$ radian

$= \frac{7\pi}{6}$ radians

c) $-60^\circ = -60 \cdot 1^\circ$

$= -60 \cdot \frac{\pi}{180}$ radians

$= -\frac{\pi}{3}$ radians

d) $100^\circ = 100 \cdot 1^\circ$

$= 100 \cdot \frac{\pi}{180}$ radians

$= \frac{5\pi}{9}$ radians

Example 2: Converting from Degrees to Radians (2 of 2)

Example 3: Converting from Radians to Degrees (1 of 2)

Convert each angle to degrees.

(a) $\frac{\pi}{4}$ radian (b) $\frac{7\pi}{6}$ radians (c) $-\frac{\pi}{3}$ radians (d) 2.5 radians

$\frac{\pi}{4}$ radian $= \frac{\pi}{4} \cdot 1$ radian

$= \frac{\pi}{4} \cdot \frac{180}{\pi}$ degrees

$= 45^\circ$.

$\left(\frac{7\pi}{6} \cdot \frac{180}{\pi}\right)^\circ = 210^\circ$

$\left(-\frac{\pi}{3} \cdot \frac{180}{\pi}\right)^\circ = -60^\circ$

$(2.5)\frac{180}{\pi}$ degrees $\simeq 143.24^\circ$

Example 3: Converting from Radians to Degrees (2 of 2)

Common Angles in Degrees and Radians

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

Objective 3

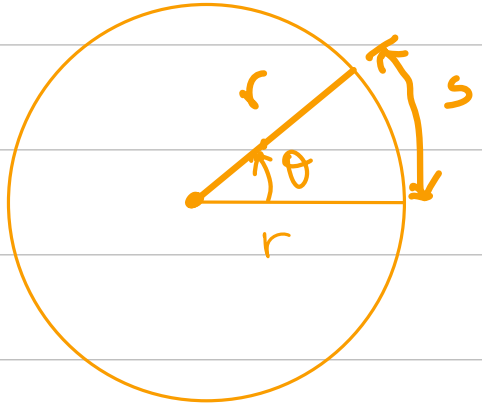
Find the Length of an Arc of a Circle

Theorem 1 — Arc Length

On a circle of radius r, a central angle of θ radians cuts out an arc whose length s satisfies

$$s = r \cdot \theta$$

$$\Rightarrow \quad r = \frac{s}{\theta} \qquad \theta = \frac{s}{r}$$



s = arc length

θ = central angle in RADIAN

Example 4: Finding the Length of an Arc of a Circle

Find the length of the arc of a circle of radius 4 meters subtended by a central angle of 0.75 radian.

$r = 4\,m$

$\theta = 0.75$ radian

$s = r \cdot \theta$

$= (4\,m)(0.75 \text{ radian})$

$s = 3\,m$

#

Objective 4

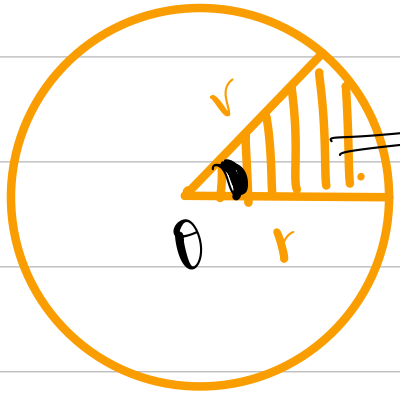
Find the Area of a Sector of a Circle

Theorem 2 — Area of a Sector

For a circle of radius r, the area A of the sector formed by a central angle of θ radians satisfies

$$A = \frac{1}{2} r^2 \cdot \theta$$

If θ and A given: $r = ?$



$$\Rightarrow \frac{2A}{\theta} = \frac{r^2 \theta}{\theta}$$

$$\Rightarrow r^2 = \frac{2A}{\theta}$$

$$\Rightarrow r = \pm\sqrt{\frac{2A}{\theta}}$$

$$r = \sqrt{\frac{2A}{\theta}}$$

Example 5: Finding the Area of a Sector of a Circle

Find the area of the sector of a circle of radius 3 meters formed by an angle of 45°. Round the answer to two decimal places.

$A = ?$

$r = 3\,m$

$\theta = 45^\circ$

$= 45 \cdot 1^\circ$

$= 45 \cdot \frac{\pi}{180}$

$= \frac{\pi}{4}$ radian.

So, $A = \frac{1}{2} r^2 \theta = \frac{1}{2}(3)^2 \cdot \left(\frac{\pi}{4}\right) = \frac{9\pi}{8} = \frac{9}{8} \cdot \frac{22}{7} = 3.53\,m^2$